Zadatak 6. Izračunaj površinu lika omeđenog

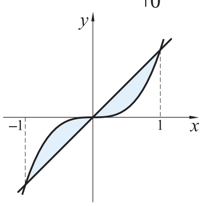
1) krivuljom $y = \ln x$ i pravcima $y = 0$, $x = e$;
2) krivuljom $xy = 3$ i pravcem $x + y = 4$;
3) krivuljom $y = x^3$ i pravcem $x - y = 0$;
4) krivuljom $y = 2x^3$ i tangentom na tu krivulju u točki $(1, 2)$;
5) krivuljama $y = \sin x$, $y = \cos x$ i pravcima $x = 0$, $x = 2\pi$;
6) krivuljom $y = x^4 - 2x^2 + 5$ i pravcima $x = 0$, $x = 1$, $y = 1$;
7) krivuljama $y = \sin\frac{\pi x}{2}$ i $y = x^2$;
8) krivuljama $y = -x^2$, $y = 2e^x$ i pravcima $x = 0$, $x = 1$;
9) krivuljama $y = \sqrt{x}$, $y = \sqrt{4-3x}$ i osi apscisa.

Rješenje. **1)** $P = \int_1^e \ln x dx = \left\{\begin{matrix} \ln x = u & dx = dv \\ \frac{dx}{x} = du & x = v \end{matrix}\right\} = x\ln x\Big|_1^e - \int_1^e dx = e\ln e - \ln 1 - e + 1 = e - e + 1 = 1$.

2) $x^2 - 4x + 3 = 0 \implies (x-1)(x-3) = 0 \implies x_1 = 1, \quad x_2 = 3$.

$P = \int_1^3 \left(-x + 4 - \frac{3}{x}\right)dx = \left(-\frac{x^2}{2} + 4x - 3\ln|x|\right)\Big|_1^3 = -\frac{9}{2} + 12 - 3\ln 3 + \frac{1}{2} - 4 + 3\ln 1 = 4 - 3\ln 3$.

3) $P = 2\int_0^1 (x - x^3)dx = 2\left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_0^1 = 2\left(\frac{1}{2} - \frac{1}{4}\right) = 2 \cdot \frac{1}{4} = \frac{1}{2}$.



4) $y' = 6x^2$, $y'(1) = 6 \implies y - y_1 = 6(x - x_1) \implies y - 2 = 6(x-1) \implies y = 6x - 4$.

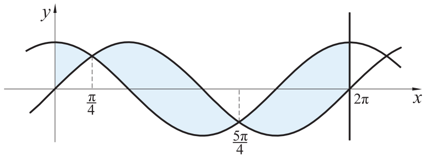
$2x^3 = 6x - 4 \implies 2x^3 - 6x + 4 = 0 \implies x^3 - 3x + 2 = 0 \implies (x+2)(x-1) = 0 \implies x_1 = -2, \quad x_2 = 1$.

$P = \int_{-2}^1 (2x^3 - 6x + 4)dx = \left(2\frac{x^4}{4} - 3x^2 + 4x\right)\Big|_{-2}^1 = \frac{1}{2}(1 - 16) - 3(1-4) + 4(1+2) = -\frac{15}{2} + 9 + 12 = \frac{42 - 15}{2} = 13\frac{1}{2}$.

5) $P = \int_0^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx + \int_{5\pi/4}^{2\pi} (\cos x - \sin x)\, dx = \sin x\Big|_0^{\pi/4} + \cos x\Big|_0^{\pi/4} - \cos x\Big|_{\pi/4}^{5\pi/4} - \sin x\Big|_{\pi/4}^{5\pi/4} + \sin x\Big|_{5\pi/4}^{2\pi} + \cos x\Big|_{5\pi/4}^{2\pi} =$

$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}Br\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) + \left(0 + \frac{\sqrt{2}}{2}\right) + \left(1 + \frac{\sqrt{2}}{2}\right) =$

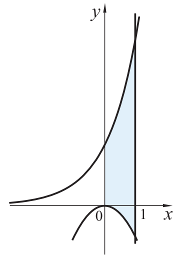
$$\sqrt{2}-1+\sqrt{2}+\sqrt{2}+\frac{\sqrt{2}}{2}+1+\frac{\sqrt{2}}{2}=4\sqrt{2}.$$



6) $P=\int_0^1(x^4-2x^2+5-1)dx=\int_0^1(x^4-2x^2+4)dx=\left(\frac{1}{5}x^5-\frac{2}{3}x^3+4x\right)\Big|_0^1$
$=\frac{1}{5}-\frac{2}{3}+4=\frac{3-10+60}{15}=\frac{53}{15}.$

7) $P=\int_0^1\left(\sin\frac{\pi x}{2}-x^2\right)dx=-\frac{2}{\pi}\cos\frac{\pi x}{2}\Big|_0^1-\frac{x^3}{3}\Big|_0^1=-\frac{2}{\pi}(0-1)-\frac{1}{3}\cdot(1-0)=$
$\frac{2}{\pi}-\frac{1}{3}.$

8) $P=\int_0^1(2e^x+x^2)dx=\left(2e^x+\frac{x^3}{3}\right)\Big|_0^1=2e+\frac{1}{3}-2=2e-\frac{5}{3}.$



9) $\sqrt{x}=\sqrt{4-3x} \implies x=4-3x \implies x=1.$
$P=\int_0^1\sqrt{x}dx+\int_1^{4/3}\sqrt{4-3x}dx=\frac{2}{3}\sqrt{x^3}\Big|_0^1-\frac{2}{9}\sqrt{(4-3x)^3}\Big|_1^{4/3}=\frac{2}{3}-\frac{2}{9}(0-$
$1)=\frac{2}{3}+\frac{2}{9}=\frac{6}{9}+\frac{2}{9}=\frac{8}{9}.$