

Zadatak 7. Izračunaj površinu lika omeđenog krivuljom $y = \ln x$, normalom na tu krivulju u točki s apscisom e , te osi apscisa.

Rješenje.

$$y = \ln x$$

$$y(e) = \ln e = 1 \implies T(e, 1)$$

$$y' = \frac{1}{x}, \quad y'(e) = \frac{1}{e} \implies k_n = -e$$

$$y - 1 = -e(x - e) \implies y = -ex + e^2 + 1 \quad \text{normala}$$

$$-ex + e^2 + 1 = 0 \implies x = \frac{e^2 + 1}{e} = e + \frac{1}{e}$$

$$\begin{aligned} P &= \int_1^e \ln x dx + \int_e^{e+\frac{1}{e}} (-ex + e^2 + 1) dx = (x \ln x - x) \Big|_1^e + \left(-\frac{e}{2}x^2 + e^2 x + x \right) \Big|_e^{e+\frac{1}{e}} \\ &= e \ln e - e - \ln 1 + 1 + \left(-\frac{e}{2} \left(e^2 + 2 + \frac{1}{e^2} \right) + e^2 \left(e + \frac{1}{e} \right) + e + \frac{1}{e} + \frac{e^3}{2} - e^3 \right) \\ &= e - e + 1 - \frac{e^3}{2} - e - \frac{1}{2e} + e^3 + e + e + \frac{1}{e} + \frac{e^3}{2} - e^3 - e = 1 + \frac{2 - 1}{2e} = 1 + \frac{1}{2e}. \end{aligned}$$

