

Zadatak 9. Nađi površinu lika omeđenog parabolom $y^2 = -x - 16$ i onim tangentama na tu parabolu koje prolaze ishodištem.

Rješenje.

$$y^2 = -x - 16$$

$$y = kx + l, \quad p = 2kl$$

$$y^2 = x = 2px \implies p = \frac{1}{2}$$

$$kl = \frac{p}{2} \implies kl = \frac{1}{4}$$

$$0 = -16k + C \implies l = 16k$$

$$16k^2 = \frac{1}{4} \implies k^2 = \frac{1}{4 \cdot 16} \implies k = \pm \frac{1}{8}, \quad l = \pm 2$$

$$\implies \text{tangente: } y = \pm \frac{1}{8}x \pm 2$$

$$\text{točka: } T\left(\frac{l}{k}, 2l\right) \implies T(16, 4)$$

$$\begin{aligned} P &= 2 \int_{-16}^{16} \left(\frac{1}{8}x + 2\right) dx - 2 \int_0^{16} \sqrt{x} dx = 2 \cdot \left(\frac{1}{16}x^2 + 2x\right) \Big|_{-16}^{16} - 2 \left(\frac{2}{3}\sqrt{x^3}\right) \Big|_0^{16} \\ &= 2(16 + 32 - 16 + 32) - \frac{4}{3} \cdot 64 = 128 - \frac{256}{3} = 128 - 85\frac{1}{3} = 42\frac{2}{3} = \frac{128}{3} = 42\frac{2}{3} \end{aligned}$$