

Zadatak 11.

Nadji površinu lika omeđenog krivuljom $y = \cos x$, $x \in [0, \frac{\pi}{2}]$, prvcima $y = 0$, $x = 0$ i tangentom na tu krivulju u točki $x = \frac{\pi}{4}$.

Rješenje.

$$T\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right), y' = -\sin x, \quad y'\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \implies y - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) \implies$$

$$y = -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2} \cdot \frac{\pi}{4} + \frac{\pi}{2} \implies y = -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}\left(1 + \frac{\pi}{4}\right) \text{ — tangenta.}$$

$$P = \int_0^{1+\pi/4} \left[-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}\left(1 + \frac{\pi}{4}\right) \right] dx - \int_0^{\pi/2} \cos x dx = \left(-\frac{\sqrt{2}}{4}x^2 + \frac{\sqrt{2}}{2}\left(1 + \frac{\pi}{4}\right)x \right) \Big|_0^{1+\pi/4} - \sin x \Big|_0^{\pi/2} = -\frac{\sqrt{2}}{4}\left(1 + \frac{\pi}{4}\right)^2 + \frac{\sqrt{2}}{2}\left(1 + \frac{\pi}{4}\right)^2 - \sin \frac{\pi}{2} =$$

$$\frac{\sqrt{2}}{2}\left(1 + \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{4}\left(1 + \frac{\pi}{4}\right)^2 - 1 = \frac{\sqrt{2}}{4}\left(1 + \frac{\pi}{4}\right)^2 - 1.$$

