

Zadatak 12. Kolika je površina lika omeđenog pravcima $x = -\frac{\pi}{4}$, $x = \frac{\pi}{4}$, krivuljom $y = \cos x$ i tangentama na tu krivulju položenu u točkama s apscisama $\frac{\pi}{4}$ i $-\frac{\pi}{4}$?

Rješenje. $y = \pm \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}\left(\frac{\pi}{4} + 1\right)$ — tangente.

$$P = 2 \int_0^{\pi/4} \left(-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}\left(\frac{\pi}{4} + 1\right) - \cos x \right) dx = 2 \left(-\frac{\sqrt{2}}{4}x^2 + \frac{\sqrt{2}}{2}\left(\frac{\pi}{4} + 1\right)x - \sin x \right) \Big|_0^{\pi/4} = 2 \left(-\frac{\sqrt{2}}{4} \cdot \frac{\pi^2}{16} + \frac{\sqrt{2}}{2}\left(\frac{\pi}{4} + 1\right)\frac{\pi}{4} - \frac{\sqrt{2}}{2} \right) = -\frac{\sqrt{2}}{32}\pi^2 + \frac{\sqrt{2}}{16}\pi^2 + \frac{\sqrt{2}}{4}\pi - \sqrt{2} = \frac{\sqrt{2}}{32}(\pi^2 + 8\pi - 32).$$

