

Zadatak 13. Nađi površinu lika omeđenog lukom krivulje $y = 2 + \sin(2x - \frac{\pi}{4})$, tangentom na tu krivulju u točki s apscisom $x = \frac{3\pi}{8}$ i pravcem $y = \frac{4}{\pi}x - \frac{1}{2}$.

Rješenje.

Tangenta je pravac $y = 3$.

$$3 = \frac{4}{\pi}x - \frac{1}{2} \implies \frac{4}{\pi}x = \frac{7}{2} \implies x = \frac{7\pi}{8}.$$

$$3 = 2 + \sin\left(2x - \frac{\pi}{4}\right) \implies 1 = \sin\left(2x - \frac{\pi}{4}\right) \implies 2x - \frac{\pi}{4} = \frac{\pi}{2} \implies 2x = \frac{3\pi}{4} \implies x = \frac{3\pi}{8}.$$

$$P = \int_{3\pi/8}^{5\pi/8} \left(3 - 2 - \sin\left(2x - \frac{\pi}{4}\right)\right) dx + \int_{5\pi/8}^{7\pi/8} \left(3 - \frac{4}{\pi}x + \frac{1}{2}\right) dx = \left(x + \cos\left(2x - \frac{\pi}{4}\right)\right) \Big|_{3\pi/8}^{5\pi/8} + \left(\frac{7}{2}x - \frac{2}{\pi}x^2\right) \Big|_{5\pi/8}^{7\pi/8} = \frac{5\pi}{8} - 1 - \frac{3\pi}{8} - 0 + \frac{7}{2} \cdot \frac{7\pi}{8} - \frac{2}{\pi} \cdot \frac{49\pi^2}{64} - \frac{7}{2} \cdot \frac{5\pi}{8} + \frac{2}{\pi} \cdot \frac{25\pi^2}{64} = \frac{\pi}{4} - 1 + \frac{49\pi}{16} - \frac{49\pi}{32} - \frac{35\pi}{16} + \frac{25\pi}{32} = \frac{8\pi + 28\pi - 24\pi}{32} - 1 = \frac{3}{8}\pi - 1.$$

