Zadatak 14. Kolika je površina lika omeđenog krivuljom $y = 1 + \cos(2x - \frac{\pi}{3})$, tangentom na krivulju u točki s apscisom $x = \frac{\pi}{6}$ te pravcem $y = \frac{4x}{\pi} - \frac{2}{3}$?

Rješenje. Tangenta je pravac $y = 2$.

$2 = 1 + \cos\left(2x - \frac{3}{\pi}\right) \implies 1 = \cos\left(2x - \frac{\pi}{3}\right) \implies 2x - \frac{\pi}{3} = 0 \implies 2x = \frac{\pi}{3} \implies x = \frac{\pi}{6}$.

$\frac{4x}{\pi} - \frac{2}{3} = 2 \implies \frac{4x}{\pi} = \frac{8}{3} \implies x = \frac{2\pi}{3}$.

$$P = \int_{\pi/6}^{5\pi/12} \left(2 - 1 - \cos\left(2x - \frac{\pi}{3}\right)\right) dx + \int_{5\pi/12}^{2\pi/3} \left(2 - \frac{4}{\pi}x + \frac{2}{3}\right) dx = \left[x - \sin\left(2x - \frac{\pi}{3}\right)\right]\Big|_{\pi/6}^{5\pi/12} + \left(\frac{8}{3}x - \frac{2}{\pi}x^2\right)\Big|_{5\pi/12}^{2\pi/3} = \frac{5\pi}{12} - 1 - \frac{\pi}{6} + 0 + \frac{8}{3}\frac{2\pi}{3} - \frac{2}{\pi}\frac{4\pi^2}{9} - \frac{8}{3}\frac{5\pi}{12} + \frac{2}{\pi}\frac{25\pi^2}{144} = \frac{5\pi}{12} - \frac{\pi}{6} + \frac{16\pi}{9} - \frac{8\pi}{9} - \frac{10\pi}{9} + \frac{25\pi}{72} - 1 = \frac{\pi}{4} - \frac{2\pi}{9} + \frac{25\pi}{72} - 1 = \frac{18 - 16 + 25}{72}\pi - 1 = \frac{27\pi}{72}\pi - 1 = \frac{3\pi}{8} - 1.$$