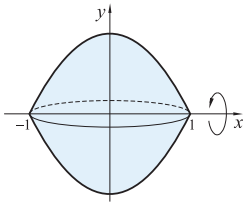
Rješenja zadataka 5.7

Zadatak 1. Izračunaj volumen rotacijskog tijela što nastaje rotacijom oko x-osi lika omeđenog krivuljama:

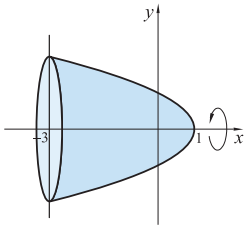
1) $y = -x^2 + 1$ i $y = 0$;
2) $y = \sqrt{1-x}$, $x = 0$ i $x = -3$;
3) $y = x^2$ i $y = \sqrt{x}$;
4) $y = \frac{1}{x}$, $y = 0$, $x = 1$ i $x = e$;
5) $y = x^3$, $y = 0$, $x = -1$ i $x = 1$;
6) $y = e^x$, $y = 0$, $x = 0$ i $x = 2$.

Rješenje.

1) $V = \pi \int_{-1}^{1} (1-x^2)^2 dx = \pi \int_{-1}^{1} (1-2x^2+x^4) dx = \pi\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right)\Big|_{-1}^{1} =$
$\pi\left(1 - \frac{2}{3} + \frac{1}{5} + 1 - \frac{2}{3} + \frac{1}{5}\right) = \pi\left(2 - \frac{4}{3} + \frac{2}{5}\right) = \pi\frac{30-20+6}{15} = \frac{16}{15}\pi.$



2) $V = \pi \int_{-3}^{0} (\sqrt{1-x})^2 dx = \pi \int_{-3}^{0} (1-x) dx = \pi\left(x - \frac{1}{2}x^2\right)\Big|_{-3}^{0} = \pi\left(3 + \frac{1}{2} \cdot 9\right) = \frac{15}{2}\pi.$



3) $V = V_1 - V_2$.
$V_1 = \pi \int_0^1 (\sqrt{x})^2 dx = \pi \int_0^1 x dx = \pi\left(\frac{x^2}{2}\right)\Big|_0^1 = \frac{\pi}{2}.$

$$V_2 = \pi \int_0^1 (x^2)^2 dx = \pi \int_0^1 x^4 dx = \pi \left(\frac{x^5}{5}\right)\Big|_0^1 = \frac{\pi}{5}.$$

$$V = V_1 - V_2 = \frac{\pi}{2} - \frac{\pi}{5} = \frac{5\pi}{10} - \frac{2\pi}{10} = \frac{3}{10}\pi.$$

4) $V = \pi \int_1^e \left(\frac{1}{x}\right)^2 dx = \pi \int_1^e \frac{dx}{x^2} = \pi\left(-\frac{1}{x}\right)\Big|_1^e = \pi\left(-\frac{1}{e} + 1\right) = \left(1 - \frac{1}{e}\right)\pi.$

5) $V = 2\pi \int_0^1 (x^3)^2 dx = 2\pi \int_0^1 x^6 dx = 2\pi\left(\frac{x^7}{7}\right)\Big|_0^1 = \frac{2}{7}\pi.$

6) $V = \pi \int_0^2 (e^x)^2 = \pi \int_0^2 e^{2x} dx = \pi \cdot \left(\frac{e^{2x}}{2}\right)\Big|_0^2 = \pi\left(\frac{e^4}{2} - \frac{1}{2}\right) = (e^4 - 1)\frac{\pi}{2}.$