**Zadatak 8.** Nađi primitivnu funkciju svake od sljedećih funkcija:

**1)** $f(x) = 3e^{2x+1} + \dfrac{1}{\cos^2 \frac{\pi x}{4}}$;

**2)** $f(x) = e^{3x+2} + \frac{1}{2}\cos\dfrac{\pi x}{4} - 3\sqrt{x}$;

**3)** $f(x) = \dfrac{3}{2x+5} - \dfrac{2}{\sin^2 \frac{x}{2}} + 4\cos 3x$;

**4)** $f(x) = \dfrac{1}{1-x} + \dfrac{e^{-x}}{2} + \sqrt{x}$;

**5)** $f(x) = 2^{2x} - \dfrac{1}{\cos^2 \frac{\pi x}{3}} + \dfrac{1}{\sqrt[3]{2x-1}}$;

**6)** $f(x) = \operatorname{ctg} x - \dfrac{1}{\cos^2(\pi - x)} + \dfrac{1}{2\sqrt{x}}e^{\sqrt{x}}$.

***Rješenje.*** **1)** $f(x) = 3e^{2x+1} + \dfrac{1}{\cos^2 \frac{\pi x}{4}}$;

$$F(x) = \int \left(3e^{2x+1} + \frac{1}{\cos^2 \frac{\pi x}{4}}\right) = 3\int e^{2x+1}dx + \int \frac{dx}{\cos^2 \frac{\pi x}{4}} = 3I_1 + I_2$$

$$I_1 = \int e^{2x+1}dx = \frac{1}{2}\int e^{2x+1}d(2x+1) = \frac{1}{2}e^{2x+1} + C_1$$

$$I_2 = \int \frac{dx}{\cos^2 \frac{\pi x}{4}} = \frac{4}{\pi}\int \frac{d(\frac{\pi x}{4})}{\cos^2 \frac{\pi x}{4}} = \frac{4}{\pi}\operatorname{tg}\frac{\pi x}{4} + C_2$$

$$\Longrightarrow F(x) = \frac{3}{2}e^{2x+1} + \frac{4}{\pi}\operatorname{tg}\frac{\pi x}{4} + C;$$

**2)** $f(x) = e^{3x+2} + \dfrac{1}{2}\cos\dfrac{\pi x}{4} - 3\sqrt{x}$;

$$F(x) = \int e^{3x+2}dx + \frac{1}{2}\int \cos\frac{\pi x}{4}dx - 3\int \sqrt{x}dx = I_1 + \frac{1}{2}I_2 - 3I_3$$

$$I_1 = \frac{1}{3}\int e^{3x+2}d(3x+2) = \frac{1}{3}e^{3x+2} + C_1$$

$$I_2 = \int \cos\frac{\pi x}{4}dx = \frac{4}{\pi}\int \cos\frac{\pi x}{4}d\left(\frac{\pi x}{4}\right) = \frac{4}{\pi}\sin\frac{\pi x}{4} + C_2$$

$$I_3 = \int \sqrt{x}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C_3 = \frac{2}{3}x\sqrt{x} + C_3$$

$$\Longrightarrow F(x) = \frac{1}{3}e^{3x+2} + \frac{2}{\pi}\sin\frac{\pi x}{4} - 2x\sqrt{x} + C;$$

**3)** $f(x) = \dfrac{3}{2x+5} - \dfrac{2}{\sin^2\frac{x}{2}} + 4\cos 3x;$

$$\begin{aligned} F(x) &= 3\int \frac{dx}{2x+5} - 2\int \frac{dx}{\sin^2\frac{x}{2}} + 4\int \cos 3x dx \\ &= \frac{3}{2}\int \frac{d(2x+5)}{2x+5} - 4\int \frac{d\left(\frac{x}{2}\right)}{\sin^2\frac{x}{2}} + \frac{4}{3}\int \cos 3x d(3x) \\ F(x) &= \frac{3}{2}\ln|2x+5| + 4\operatorname{ctg}\frac{x}{2} + \frac{4}{3}\sin 3x + C; \end{aligned}$$

**4)** $f(x) = \dfrac{1}{1-x} + \dfrac{e^{-x}}{2} + \sqrt{x};$

$$\begin{aligned} F(x) &= \int \frac{dx}{1-x} + \frac{1}{2}\int e^{-x}dx + \int \sqrt{x}dx \\ &= -\int \frac{d(x-1)}{x-1} - \frac{1}{2}\int e^{-x}d(-x) + \int x^{\frac{1}{2}}dx \\ F(x) &= -\ln|x-1| - \frac{1}{2}e^{-x} + \frac{2}{3}x\sqrt{x} + C; \end{aligned}$$

**5)** $f(x) = 2^{2x} - \dfrac{1}{\cos^2\frac{\pi x}{3}} + \dfrac{1}{\sqrt[3]{2x-1}};$

$$F(x) = \int 2^{2x}dx - \int \frac{dx}{\cos^2\frac{\pi x}{3}} + \int \frac{dx}{\sqrt[3]{2x-1}} = \frac{1}{2}\int 2^{2x}d(2x) - \frac{3}{\pi}\int \frac{d\left(\frac{\pi x}{3}\right)}{\cos^2\frac{\pi x}{3}}$$

$$+\frac{1}{2}\int \frac{d(2x-1)}{\sqrt[3]{2x-1}} = \frac{1}{2\ln 2}2^{2x} - \frac{3}{\pi}\operatorname{tg}\frac{\pi x}{3} + \frac{1}{2}\int (2x-1)^{-\frac{1}{3}}d(2x-1)$$

$$\Longrightarrow F(x) = \frac{2^{2x-1}}{\ln 2} - \frac{3}{\pi}\operatorname{tg}\frac{\pi x}{3} + \frac{3}{4}\sqrt[3]{(2x-1)^2} + C;$$

**6)** $f(x) = \operatorname{ctg} x - \dfrac{1}{\cos^2(\pi - x)} + \dfrac{1}{2\sqrt{x}}e^{\sqrt{x}};$

$$F(x) = \int \operatorname{ctg} x dx - \int \frac{dx}{\cos^2(\pi-x)} + \int \frac{e^{\sqrt{x}}}{2\sqrt{x}}dx = I_1 - I_2 + I_3$$

$$I_1 = \int \frac{\cos x dx}{\sin x} = \left\{ \begin{array}{c} \sin x = t \\ \cos x dx = dt \end{array} \right\} = \int \frac{dt}{t} = \ln|t| + C_1 = \ln|\sin x| + C$$

$$\begin{aligned} I_2 &= \int \frac{dx}{\cos^2(\pi-x)} = -\int \frac{d(\pi-x)}{\cos^2(\pi-x)} = -\operatorname{tg}(\pi-x) + C_2 \\ &= \operatorname{tg}(x-\pi) + C_2 = \operatorname{tg} x + C_2 \end{aligned}$$

$$I_3 = \int \frac{e^{\sqrt{x}}}{2\sqrt{x}}dx = \left\{ \begin{array}{c} \sqrt{x} = t \\ \dfrac{dx}{2\sqrt{x}} = dr \end{array} \right\} = \int e^t dt = e^t + C_3 = e^{\sqrt{x}} + C_3$$

$$\Longrightarrow F(x) = \ln|\sin x| - \operatorname{tg} x + e^{\sqrt{x}} + C.$$