**Zadatak 9.** Za danu funkciju $f(x)$ nađi primitivnu funkciju $F(x)$ uz dani uvjet:

**1)** $f(x) = 3x\sqrt{x^2+1}$, $F(1) = \sqrt{2}$;

**2)** $f(x) = \cos\left(\frac{x}{2} + \frac{\pi}{3}\right)$, $F(\pi) = 1$;

**3)** $f(x) = \frac{2x+1}{x^2+x}$, $F(1) = \ln 2$.

**4)** $f(x) = \sin x \cos x$, $F(\frac{\pi}{6}) = -\frac{1}{4}$;

**5)** $f(x) = \frac{x}{x-1}$, $F(2) = 2$;

**6)** $f(x) = \sqrt{2x+3}$, $F(-1) = 1$;

**7)** $f(x) = 2e^{-x} + 1$, $F(0) = 2$;

**8)** $f(x) = 3\cos\left(2x - \frac{\pi}{6}\right)$, $F(\frac{\pi}{2}) = \frac{3}{4}$.

***Rješenje.*** **1)** $f(x) = 3x\sqrt{x^2+1}$, $F(1) = \sqrt{2}$;

$$F(x) = 3\int x\sqrt{x^2+1}dx = \left\{\begin{array}{c} x^2+1 = t^2 \\ 2xdx = 2tdt \\ xdx = tdt \end{array}\right\} = 3\int t^2 dt = t^3 + C$$

$$F(x) = \sqrt{(x^2+1)^3 + C}$$

$$F(1) = 2\sqrt{2} + C = \sqrt{2} \implies C = -\sqrt{2}$$

$$\implies F(x) = (x^2+1)\sqrt{x^2+1} - \sqrt{2};$$

**2)** $f(x) = \cos\left(\frac{x}{2} + \frac{\pi}{3}\right)$, $F(\pi) = 1$;

$$F(x) = \int \cos\left(\frac{x}{2} + \frac{\pi}{3}\right)dx = 2\int \cos\left(\frac{x}{2} + \frac{\pi}{3}\right)d\left(\frac{x}{2} + \frac{\pi}{3}\right)$$

$$F(x) = 2\sin\left(\frac{x}{2} + \frac{\pi}{3}\right) + C$$

$$F(\pi) = 2\sin\frac{5\pi}{6} + C = 1 + C = 1 \implies C = 0$$

$$\implies F(x) = 2\sin\left(\frac{x}{2} + \frac{\pi}{3}\right);$$

**3)** $f(x) = \frac{2x+1}{x^2+x}$, $F(1) = \ln 2$;

$$F(x) = \int \frac{2x+1}{x^2+x}dx = \int \frac{d(x^2+x)}{x^2+x} = \ln|x^2+x| + C$$

$$F(1) = \ln 2 + C = \ln 2 \implies C = 0$$

$$\implies F(x) = \ln|x^2+x|;$$

**4)** $f(x) = \sin x \cos x$, $F(\frac{\pi}{6}) = -\frac{1}{4}$;

$$F(x) = \int \sin x \cos x dx = \int \sin x d(\sin x) = \frac{1}{2}\sin^2 x + C_1$$

$$= \frac{1}{2}\frac{1-\cos 2x}{2} + C_1 = \frac{1}{4} - \frac{1}{4}\cos 2x + C_1 = -\frac{1}{4}\cos 2x + C_2$$

$$F\left(\frac{\pi}{6}\right) = -\frac{1}{4}\cos\frac{\pi}{3} + C = -\frac{1}{4}\cdot\frac{1}{2} + C = -\frac{1}{8} + C = -\frac{1}{4} \implies C = -\frac{1}{8}$$

$$\implies F(x) = -\frac{1}{4}\cos 2x - \frac{1}{8};$$

**5)** $f(x) = \frac{x}{x-1}$, $F(2) = 2$;

$$F(x) = \int \frac{x}{x-1}dx = \int \frac{x-1+1}{x-1}dx = \int \left(1 + \frac{1}{x-1}\right)dx$$

$$F(x) = x + \ln|x-1| + C$$

$$F(2) = 2 + C = 2 \implies C = 0$$

$$\implies F(x) = x + \ln|x-1|;$$

**6)** $f(x) = \sqrt{2x+3}$, $F(-1) = 1$;

$$F(x) = \int \sqrt{2x+3}dx = \left\{\begin{array}{c} 2x+3 = t^2 \\ 2dx = 2tdt \\ dx = tdt \end{array}\right\} = \int t^2 dt = \frac{t^3}{3} + C$$

$$F(x) = \frac{(2x+3)\sqrt{2x+3}}{3} + C$$

$$F(-1) = \frac{1}{3} + C = 1 \implies C = \frac{2}{3}$$

$$\implies F(x) = \frac{1}{3}(2x+3)\sqrt{2x+3} + \frac{2}{3};$$

**7)** $f(x) = 2e^{-x} + 1$, $F(0) = 2$;

$$F(x) = \int (2e^{-x} + 1)dx = 2\int e^{-x}dx + \int dx = -2e^{-x} + x + C$$

$$F(0) = -2 + C = 2 \implies C = 4$$

$$\implies F(x) = x - 2e^{-x} + 4;$$

**8)** $f(x) = 3\cos\left(2x - \frac{\pi}{6}\right)$, $F(\frac{\pi}{2}) = \frac{3}{4}$;

$$F(x) = 3\int \cos\left(2x - \frac{\pi}{6}\right) = \frac{3}{2}\int \cos\left(2x - \frac{\pi}{6}\right)d\left(2x - \frac{\pi}{6}\right)$$

$$F(x) = \frac{3}{2}\left(2x - \frac{\pi}{6}\right) + C$$

$$F\left(\frac{\pi}{2}\right) = \frac{3}{2}\sin\frac{5\pi}{6} + C = \frac{3}{4} + C = \frac{3}{4} \implies C = 0$$

$$\implies F(x) = \frac{3}{2}\sin\left(2x - \frac{\pi}{6}\right).$$