



Zadatak 11. Izračunaj integrale:

$$1) \int \sin \frac{1}{x} \cdot \frac{1}{x^2} dx;$$

$$2) \int \sin^3 x \cos^3 x dx;$$

$$3) \int \cos^5 x \sin^4 x dx;$$

$$4) \int \cos^5 x \sqrt{\sin x} dx.$$

Rješenje.

$$1) \int \sin \frac{1}{x} \cdot \frac{1}{x^2} dx = \left\{ \begin{array}{l} \frac{1}{x} = t \\ \frac{dx}{x^2} = dt \end{array} \right\} = - \int \sin t dt = \cos t + C = \cos \frac{1}{x} + C;$$

$$2) \int \sin^3 x \cos^3 x dx = \int \sin^2 x (1 - \sin^2 x) \cos x dx = \int (\sin^3 x - \sin^5 x) d(\sin x) = \frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + C;$$

$$3) \int \cos^5 x \sin^4 x dx = \int (1 - \sin^2 x)^2 \sin^4 x d(\sin x) = \int (\sin^4 x - 2 \sin^6 x + \sin^8 x) d(\sin x) = \frac{1}{5} \sin^5 x - \frac{2}{7} \sin^7 x + \frac{1}{9} \sin^9 x + C;$$

4)

$$\begin{aligned} \int \cos^5 x \sqrt{\sin x} dx &= \int (1 - \sin^2 x)^2 \sin^{\frac{1}{2}} x d(\sin x) = \left\{ \begin{array}{l} \sin x = t \\ d(\sin x) = dt \end{array} \right\} \\ &= \int (1 - t^2)^2 t^{\frac{1}{2}} dt = \int (1 - 2t^2 + t^4) t^{\frac{1}{2}} dt \\ &= \int (t^{\frac{1}{2}} - 2t^{\frac{5}{2}} + t^{\frac{9}{2}}) dt = \frac{2}{3} t^{\frac{3}{2}} - \frac{4}{7} t^{\frac{7}{2}} + \frac{2}{11} t^{\frac{11}{2}} + C \\ &= t^{\frac{1}{2}} \left(\frac{2}{3} t - \frac{4}{7} t^3 + \frac{2}{11} t^5 \right) + C \\ &= \left(\frac{2}{3} \sin x - \frac{4}{7} \sin^3 x + \frac{2}{11} \sin^5 x \right) \sqrt{\sin x} + C. \end{aligned}$$