

**Zadatak 13.** Riješi jednadžbe:

$$1) \int_0^t \cos(x + t^2) dx = \sin t, \quad t \in [2, 3];$$

$$2) \int_{-t}^t \cos(x + 2t^2 - t) dx = -\sin 2t, \\ t \in \left[-\frac{3}{2}, -\frac{1}{2}\right];$$

$$3) \int_0^{2t} \sin(x - t^2) dx = \sin 2t, \quad t \in [2, 3];$$

$$4) \int_{\frac{\pi}{2}}^t \sin(x - t^2) dx = -\cos t, \quad t \in \left[-\frac{7}{2}, -\frac{5}{2}\right];$$

$$5) \int_{\frac{\pi}{2}}^t \sin x dx = \sin 2t, \quad t \in [0, 2\pi].$$

**Rješenje.**

$$1) \int_0^t \cos(x + t^2) dx = \left\{ \begin{array}{l} v = x + t^2 \\ dv = dx \end{array} \right\} = \int_{t^2}^{t+t^2} \cos v dv = \sin v \Big|_{t^2}^{t+t^2} = \sin(t^2 + t) - \sin t^2.$$

$$\sin(t^2 + t) - \sin t^2 = \sin t$$

$$2) \int_{-t}^t \cos(x + 2t^2 - t) dx = \left\{ \begin{array}{l} v = x + 2t^2 - t \\ dv = dx \end{array} \right\} = \int_{2t^2-2t}^{2t^2} \cos v dv = \sin v \Big|_{2t^2-2t}^{2t^2} = \sin 2t^2 - \sin(2t^2 - 2t).$$

$$\sin 2t^2 - \sin(2t^2 - 2t) = -\sin 2t$$

$$2 \cos \frac{4t^2 - 2t}{2} \sin \frac{2t}{2} = -2 \sin t \cos t$$

$$\cos(2t^2 - 1) = -\cos t$$

$$3) \int_0^{2t} \sin(x - t^2) dx = \left\{ \begin{array}{l} v = x - t^2 \\ dv = dx \end{array} \right\} = \int_{-t^2}^{2t-t^2} \sin v dv = -\cos v \Big|_{-t^2}^{2t-t^2} = -(\cos(2t - t^2) - \cos(-t^2)) = -\cos(2t - t^2) + \cos t^2 = -2 \sin t \sin(t^2 - t).$$

$$-2 \sin t \sin(t^2 - t) = \sin 2t$$

$$-2 \sin t \sin(t^2 - t) = 2 \sin t \cos t$$

$$-\sin(t^2 - t) = \cos t$$

$$4) \int_{\frac{\pi}{2}}^t \sin(x - t^2) dx = \left\{ \begin{array}{l} v = x - t^2 \\ dv = dx \end{array} \right\} = \int_{\frac{\pi}{2}-t^2}^{t-t^2} \sin v dv = -\cos v \Big|_{\frac{\pi}{2}-t^2}^{t-t^2} = -(\cos(t - t^2) + \cos(\frac{\pi}{2} - t^2)) = -\cos(t - t^2) + \sin t^2.$$

$$-\cos(t - t^2) + \sin t^2 = -\cos t$$

$$5) \int_{\frac{\pi}{2}}^t \sin x \, dx = -\cos x \Big|_{\frac{\pi}{2}}^t = -\cos t + \cos \frac{\pi}{2} = -\cos t.$$

$$-\cos t = \sin 2t$$

$$2 \sin t \cos t + \cos t = 0$$

$$\cos t(2 \sin t + 1) = 0$$

$$t = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}.$$