

Zadatak 15. Za koje je $a \in \mathbf{R}$ ispunjeno

- 1) $\int_0^a (2 - 4x + 3x^2) dx \leq a$;
- 2) $\int_1^2 (a^2 + 4(1 - a)x + 4x^3) dx \leq 12$;
- 3) $\int_1^a (a - 4x) dx \geq 6 - 5a$;
- 4) $\int_0^a x dx \leq a + 4$?

Rješenje. 1) $\int_0^a (2 - 4x + 3x^2) dx = 2 \int_0^a dx - 4 \int_0^a x dx + 3 \int_0^a x^2 dx = 2x \Big|_0^a - \frac{4x^2}{2} \Big|_0^a + \frac{3x^3}{3} \Big|_0^a = 2a - 2a^2 + a^3.$

$$2a - 2a^2 + a^3 \leq a$$

$$a^3 - 2a^2 + a \leq 0$$

$$a^2(a - 1) - a(a - 1) \leq 0$$

$$a(a - 1)^2 \leq 0$$

$$a < 0, a = 1$$

2) $\int_1^2 (a^2 + 4(1 - a)x + 4x^3) dx = a^2x \Big|_1^2 + 4(1 - a) \cdot \frac{x^2}{2} \Big|_1^2 + 4 \cdot \frac{x^4}{4} \Big|_1^2 = a^2 + 2(1 - a) \cdot 3 + 15 = a^2 + 6 - 6a + 15 = a^2 - 6a + 21.$

$$a^2 - 6a + 21 \leq 12$$

$$a^2 - 6a + 9 \leq 0$$

$$(a - 3)^2 \leq 0$$

$$a = 3$$

3) $\int_1^a (a - 4x) dx = ax \Big|_1^a - 4 \cdot \frac{x^2}{2} \Big|_1^a = a(a - 1) - 2(a^2 - 1) = a^2 - a - 2a^2 + 2 = -a^2 - a + 2.$

$$-a^2 - a + 2 \geq 6 - 5a$$

$$-a^2 - a + 5a + 2 - 6 \geq 0$$

$$-a^2 + 4a - 4 \geq 0$$

$$a^2 - 4a + 4 \leq 0$$

$$(a - 2) \leq 0$$

$$a = 2$$

$$4) \int_0^a x dx = \frac{x^2}{2} \Big|_0^a = \frac{1}{2}(a^2 - 0) = \frac{a^2}{2}.$$

$$\frac{a^2}{2} \leq a + 4$$

$$a^2 - 2a - 8 \leq 0$$

$$(a - 4)(a + 2) \leq 0$$

$$a \in [-2, 4]$$