

Zadatak 19.

Odredi brojeve a i b tako da

- 1) funkcija $f(x) = a \sin \pi x + b$ zadovoljava
uvjet $f'(1) = 2$, $\int_0^2 f(x)dx = 4$;
- 2) funkcija $f(x) = a \sin 2x + b$ zadovoljava
uvjet $f'(0) = 4$, $\int_0^{2\pi} f(x)dx = 3$;
- 3) funkcija $f(x) = a \cdot 3^x + b$ zadovoljava
uvjet $f'(0) = 2$, $\int_1^2 f(x)dx = 12$;
- 4) funkcija $f(x) = a \cdot 2^x + b$ zadovoljava
uvjet $f'(1) = 2$, $\int_0^3 f(x)dx = 7$.

Rješenje.

1) $f'(x) = a\pi \cos \pi x \implies f'(1) = a\pi \cos \pi = -a\pi = 2 \implies a = -\frac{2}{\pi}$.

$$\int_0^2 (a \sin \pi x + b)dx = -\frac{a}{\pi} \cos \pi x \Big|_0^2 + bx \Big|_0^2 = -\frac{a}{\pi}(\cos 2\pi - \cos 0) + 2b = 2b = 4 \implies b = 2.$$

2) $f'(x) = 2a \cos 2x \implies f'(0) = 2a = 4 \implies a = 2$.

$$\int_0^{2\pi} (a \sin 2x + b)dx = -\frac{a}{2} \cos 2x \Big|_0^{2\pi} + bx \Big|_0^{2\pi} = -\frac{a}{2}(\cos 4\pi - \cos 0) + 2\pi b = 2\pi b = 3 \implies b = \frac{3}{2\pi}.$$

3) $f'(x) = a \cdot 3^x \cdot \ln 3 \implies f'(0) = a \ln 3 = 2 \implies a = \frac{2}{\ln 3}$.

$$\int_1^2 (a \cdot 3^x + b)dx = a \cdot \frac{3^x}{\ln 3} \Big|_1^2 + bx \Big|_1^2 = \frac{a}{\ln 3}(9 - 3) + b(2 - 1) = \frac{6a}{\ln 3} + b = \frac{12}{\ln^2 3} + b = 12 \implies b = 12 - \frac{12}{\ln^2 3}.$$

4) $f'(x) = a \cdot 2^x \cdot \ln 2 \implies f'(1) = a \cdot 2 \ln 2 = 2 \implies a = \frac{1}{\ln 2}$.

$$\int_0^3 (a \cdot 2^x + b)dx = \frac{a \cdot 2^x}{\ln 2} \Big|_0^3 + bx \Big|_0^3 = \frac{a}{\ln 2}(8 - 1) + b \cdot 3 = \frac{7a}{\ln 2} + 3b = \frac{7}{\ln^2 2} + 3b = 7 \implies 3b = 7 - \frac{7}{\ln^2 2} = 7 \left(1 - \frac{1}{\ln^2 2}\right) \implies b = \frac{7}{3} \left(1 - \frac{1}{\ln^2 2}\right).$$