Zadatak 23. Nađi površinu lika omeđenog:

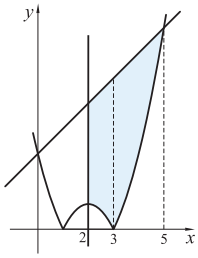
1) krivuljom $y = |x^2 - 4x + 3|$ i pravcima $x = 2$, $y = x + 3$;

2) krivuljom $y = |x^2 - 4x + 3|$ i pravcem $y = x + 3$;

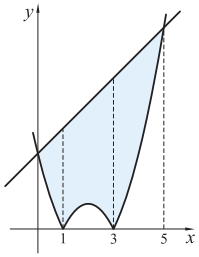
3) krivuljama $y = \big||x| - 1\big|$ i $y = -x^2 + 1$;

4) krivuljama $y = x^2 + 1$, $y = \big||x| - 1\big|$, $|x| = 1$.

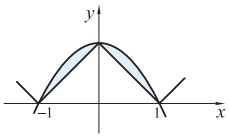
Rješenje. **1)** $P = \int_2^3 (x + 3 + x^2 - 4x + 3)dx + \int_3^5 (x + 3 - x^2 + 4x - 3)dx =$
$\int_2^3 (x^2 - 3x + 6)dx + \int_3^5 (-x^2 + 5x)dx = \frac{x^3}{3}\Big|_2^3 - \frac{3x^2}{2}\Big|_2^3 + 6x\Big|_2^3 - \frac{x^3}{3}\Big|_3^5 + \frac{5x^2}{2}\Big|_3^5 =$
$\frac{1}{3}(27 - 8) - \frac{3}{2}(9 - 4) + 6 - \frac{1}{3}(125 - 27) + |dr52(25 - 9) = \frac{19}{3} - \frac{15}{2} + 6 -$
$\frac{98}{3} + 40 = \frac{-158 - 45 + 276}{6} = \frac{73}{6}$.



2) $P=\int_0^1(x+3-x^2+4x-3)dx+\int_1^3(x+3+x^2-4x+3)dx+\int_3^5(x+3-x^2+4x-3)dx=\int_0^1(-x^2+5x)dx+\int_1^3(x^2-3x+6)dx+\int_3^5(5x/x^2)dx=-\frac{x^3}{3}\Big|_0^1+\frac{5x^2}{2}\Big|_0^1+\frac{x^3}{3}\Big|_1^3+\frac{3x^2}{2}\Big|_1^3+6x\Big|_1^3-\frac{x^3}{3}\Big|_3^5+\frac{5x^2}{2}\Big|_3^5=-\frac{1}{3}+\frac{5}{2}+\frac{27}{3}-\frac{1}{3}-\frac{3}{2}(9-1)+6\cdot 2-\frac{1}{3}(125-27)+\frac{5}{2}(25-9)=\frac{25}{3}+\frac{5}{2}-12+12-\frac{98}{3}+40=-\frac{73}{3}+\frac{5}{2}+40=\frac{-146+15+240}{6}=\frac{109}{6}.$



3) $P=2\int_0^1(-x^2+1+x-1)dx=2\int_0^1(-x^2+x)dx=2\left[-\frac{x^3}{3}\Big|_0^1+\frac{x^2}{2}\Big|_0^1\right]=2\left(-\frac{1}{3}+\frac{1}{2}\right)=2\cdot\frac{-2+3}{6}=\frac{1}{3}.$



4) $P=2\int_0^1(x^2+1+x-1)dx=2\int_0^1(x^2+x)dx=2\left[\frac{x^3}{3}\Big|_0^1+\frac{x^2}{2}\Big|_0^1\right]=2\left(\frac{1}{3}+\frac{1}{2}\right)=2\cdot\frac{2+3}{6}=\frac{5}{3}.$