

Zadatak 27. Kolika je površina lika omeđenog krivuljom $y = \frac{3}{4 - |x - 2|}$, tangentom na tu krivulju u točki s apscisom $x = 1$ i pravcima $x = 3$ i $x = 4$?

Rješenje.

$$y(1) = \frac{3}{4 - |1 - 2|} = \frac{3}{4 - |-1|} = \frac{3}{3} = 1.$$

$$y' = \left(\frac{3}{4 + x - 2} \right)' = \left(\frac{3}{2 + x} \right)' = -\frac{3}{(2 + x)^2} \implies y'(1) = -\frac{3}{9} = -\frac{1}{3}.$$

Tangenta je $y - 1 = -\frac{1}{3}(x - 1) \implies y = -\frac{1}{3}x + \frac{1}{3} + 1 \implies y = -\frac{1}{3}x + \frac{4}{3}.$

$$P = \int_3^4 \left(\frac{3}{4 - x + 2} + \frac{1}{3}x - \frac{4}{3} \right) dx = \int_3^4 \frac{3}{6 - x} dx + \frac{1}{3} \cdot \frac{x^2}{2} \Big|_3^4 - \frac{4}{3}x \Big|_3^4 = -3 \ln |6 - x| \Big|_3^4 + \frac{1}{6}(16 - 9) - \frac{4}{3} = -3(\ln 2 - \ln 3) + \frac{7}{6} - \frac{8}{6} = 3(\ln 3 - \ln 2) - \frac{1}{6} = 3 \ln \frac{3}{2} - \frac{1}{6}.$$

