

Zadatak 32. Izračunaj volumen rotacijskog tijela što se dobije rotacijom sinusoide oko osi apscisa unutar intervala $[0, \pi]$.

Rješenje.

$$\begin{aligned} V &= \pi \int_0^{\pi} \sin^2 x dx = \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} dx = \pi \left(\int_0^{\pi} \frac{1}{2} dx - \frac{1}{2} \int_0^{\pi} \cos 2x dx \right) = \\ &= \pi \left(\frac{\pi}{2} - \frac{1}{2} \int_0^{2\pi} \frac{\cos t}{2} dt \right) = \pi \left(\frac{\pi}{2} - \frac{1}{4} \sin t \Big|_0^{2\pi} \right) = \pi \left(\frac{\pi}{2} - \frac{1}{4} (\sin 2\pi - \sin 0) \right) = \\ &= \frac{\pi^2}{2}. \end{aligned}$$