

Zadatak 33. Izračunaj volumen tijela nastalog rotacijom krivulje $y = \cos x$ i pravcima $x = 0$ i $x = \frac{\pi}{2}$ oko osi apscisa.

Rješenje.

$$V = \pi \int_0^{\frac{\pi}{2}} \cos^2 x dx = \pi \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2x}{2} dx = \pi \left(\frac{1}{2} \int_0^{\frac{\pi}{2}} dx + \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos 2x dx \right) =$$
$$\pi \left(\frac{\pi}{4} + \frac{1}{2} \int_0^{\pi} \frac{\cos t}{2} dt \right) = \pi \left(\frac{\pi}{4} - \frac{1}{4} \sin t \Big|_0^{\pi} \right) = \pi \left(\frac{\pi}{4} - \frac{1}{4} (\sin \pi - \sin 0) \right) = \frac{\pi^2}{4}.$$