Zadatak 2. Napiši prvih pet članova niza (a_n) ako je:

1) $a_n = \lfloor \sqrt{n} \rfloor$; **2)** $a_n = \lfloor \sqrt{n^2+n} \rfloor$;
3) $a_n = \log 0.1^n$; **4)** $a_n = \lfloor \log_2 n \rfloor$;
5) $a_n = \sin \pi n$; **6)** $a_n = \cos \dfrac{2\pi n}{3}$.

Rješenje. **1)** $a_n = \lfloor \sqrt{n} \rfloor$;

$$a_1 = \lfloor \sqrt{1} \rfloor = 1, \qquad a_2 = \lfloor \sqrt{2} \rfloor = \lfloor 1.41\ldots \rfloor = 1,$$
$$a_3 = \lfloor \sqrt{3} \rfloor = \lfloor 1.73\ldots \rfloor = 1, \quad a_4 = \lfloor \sqrt{4} \rfloor = \lfloor 2 \rfloor = 2,$$
$$a_5 = \lfloor \sqrt{5} \rfloor = \lfloor 2.236\ldots \rfloor = 2;$$

2) $a_n = \lfloor \sqrt{n^2+n} \rfloor$;

$$a_1 = \lfloor \sqrt{1^2+1} \rfloor = \lfloor \sqrt{2} \rfloor = 1, \quad a_2 = \lfloor \sqrt{2^2+2} \rfloor = \lfloor \sqrt{6} \rfloor = 2,$$
$$a_3 = \lfloor \sqrt{3^2+3} \rfloor = \lfloor \sqrt{12} \rfloor = 3, \quad a_4 = \lfloor \sqrt{4^2+4} \rfloor = \lfloor \sqrt{20} \rfloor = 4,$$
$$a_5 = \lfloor \sqrt{5^2+5} \rfloor = \lfloor \sqrt{30} \rfloor = 5;$$

3) $a_n = \log 0.1^n = \log(10^{-1})^n = \log 10^{-n} = -n$;

$$a_1 = -1, \quad a_2 = -2, \quad a_3 = -3, \quad a_4 = -4, \quad a_5 = -5;$$

4) $a_n = \lfloor \log_2 n \rfloor$;

$$a_1 = \lfloor \log_2 1 \rfloor = 0, \quad a_2 = \lfloor \log_2 2 \rfloor = 1, \quad a_3 = \lfloor \log_2 3 \rfloor = 1,$$
$$a_4 = \lfloor \log_2 4 \rfloor = 2, \quad a_5 = \lfloor \log_2 5 \rfloor = 2;$$

5) $a_n = \sin \pi n$; $\quad \sin(n \cdot \pi) = 0,\ n \in \mathbf{N} \implies$

$$a_1 = a_2 = a_3 = a_4 = a_5 = \ldots = 0;$$

6) $a_n = \cos \dfrac{2\pi n}{3}$;

$$a_1 = \cos \frac{2\pi}{3} = -\frac{1}{2}, \qquad a_2 = \cos \frac{4\pi}{3} = -\frac{1}{2},$$
$$a_3 = \cos(2\pi) = 1, \qquad a_4 = \cos \frac{8\pi}{3} = \cos \frac{2\pi}{3} = -\frac{1}{2},$$
$$a_5 = \cos \frac{10\pi}{3} = \cos \frac{4\pi}{3} = -\frac{1}{2}.$$