

Zadatak 3. Napiši prvih pet članova niza (a_n) kojem je opći član a_n :

$$1) a_n = \sum_{k=1}^n k; \quad 2) a_n = \sum_{k=1}^n k(k+1);$$

$$3) a_n = \sum_{k=1}^n \frac{1}{k(k+1)}; \quad 4) a_n = \sum_{k=1}^n \frac{k}{k+1}.$$

Rješenje. 1) $a_n = \sum_{k=1}^n k;$

$$a_1 = 1, \quad a_2 = 1 + 2 = 3, \quad a_3 = 1 + 2 + 3 = 6,$$

$$a_4 = 1 + 2 + 3 + 4 = 10, \quad a_5 = 1 + 2 + 3 + 4 + 5 = 15;$$

$$2) a_n = \sum_{k=1}^n k(k+1) = \sum_{k=1}^{n-1} k(k+1) + n(n+1) = a_{n-1} + n(n+1);$$

$$a_1 = 1(1+1) = 2,$$

$$a_2 = a_1 + 2(2+1) = 2 + 6 = 8,$$

$$a_3 = a_2 + 3(3+1) = 8 + 12 = 20,$$

$$a_4 = a_3 + 4(4+1) = 20 + 20 = 40,$$

$$a_5 = a_4 + 5(5+1) = 40 + 30 = 70;$$

$$3) a_n = \sum_{k=1}^n \frac{1}{k(k+1)} = \sum_{k=1}^{n-1} \frac{1}{k(k+1)} + \frac{1}{n(n+1)} = a_{n-1} + \frac{1}{n(n+1)};$$

$$a_1 = \frac{1}{1(1+1)} = \frac{1}{2},$$

$$a_2 = a_1 + \frac{1}{2(2+1)} = \frac{1}{2} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3},$$

$$a_3 = a_2 + \frac{1}{3(3+1)} = \frac{2}{3} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4},$$

$$a_4 = a_3 + \frac{1}{4(4+1)} = \frac{3}{4} + \frac{1}{20} = \frac{16}{20} = \frac{4}{5},$$

$$a_5 = a_4 + \frac{1}{5(5+1)} = \frac{4}{5} + \frac{1}{30} = \frac{25}{30} = \frac{5}{6};$$

$$4) a_n = \sum_{k=1}^n \frac{k}{k+1} = \sum_{k=1}^{n-1} \frac{k}{k+1} + \frac{n}{n+1} = a_{n-1} + \frac{n}{n+1};$$

$$a_1 = \frac{1}{1+1} = \frac{1}{2},$$

$$a_2 = a_1 + \frac{2}{2+1} = \frac{1}{2} + \frac{2}{3} = \frac{7}{6},$$

$$a_3 = a_2 + \frac{3}{3+1} = \frac{7}{6} + \frac{3}{4} = \frac{23}{12},$$

$$a_4 = a_3 + \frac{4}{4+1} = \frac{23}{12} + \frac{4}{5} = \frac{163}{60},$$

$$a_5 = a_4 + \frac{5}{5+1} = \frac{163}{60} + \frac{5}{6} = \frac{213}{60} = \frac{71}{20}.$$