Zadatak 4. Napiši prvih pet članova niza zadanog općim članom

1) $a_n = 2^n - 1$;

2) $a_n = 3^{-n}$;

3) $a_n = 1 + (-1)^n$;

4) $a_n = \dfrac{n+1}{n}$;

5) $a_n = \left(1 + \dfrac{1}{n}\right)^n$;

6) $a_n = 1 + \dfrac{1}{1!} + \ldots + \dfrac{1}{n!}$.

Rješenje. **1)** $a_n = 2^n - 1$;

$$a_1 = 2 - 1 = 1, \quad a_2 = 4 - 1 = 3, \quad a_3 = 8 - 1 = 7,$$
$$a_4 = 16 - 1 = 15, \quad a_5 = 32 - 1 = 31$$

2) $a_n = 3^{-n} = \dfrac{1}{3^n}$;

$$a_1 = \frac{1}{3}, \quad a_2 = \frac{1}{3^2} = \frac{1}{9}, \quad a_3 = \frac{1}{3^3} = \frac{1}{27},$$
$$a_4 = \frac{1}{3^4} = \frac{1}{81}, \quad a_5 = \frac{1}{3^5} = \frac{1}{243};$$

3) $a_n = 1 + (-1)^n$;

$$\Longrightarrow \; a_1 = 0,\ a_2 = 2,\ a_3 = 0,\ a_4 = 2,\ a_5 = 0;$$

4) $a_n = \dfrac{n+1}{n}$;

$$\Longrightarrow \; a_1 = \frac{2}{1},\ a_2 = \frac{3}{2},\ a_3 = \frac{4}{3},\ a_4 = \frac{5}{4},\ a_5 = \frac{6}{5};$$

5) $a_n = \left(1 + \dfrac{1}{n}\right)^n = \left(\dfrac{n+1}{n}\right)^n$;

$$a_1 = \frac{2^1}{1^1} = 2, \quad a_2 = \frac{3^2}{2^2} = \frac{9}{4}, \quad a_3 = \frac{4^3}{3^3} = \frac{64}{27},$$
$$a_4 = \frac{5^4}{4^4} = \frac{625}{256}, \quad a_5 = \frac{6^5}{5^5} = \frac{7776}{3125};$$

6) $a_n = 1 + \dfrac{1}{1!} + \ldots + \dfrac{1}{(n-1)!} + \dfrac{1}{n!} = a_{n-1} + \dfrac{1}{n!}$.

$$a_1 = 1 + \frac{1}{1!} = 2, \qquad a_2 = a_1 + \frac{1}{2!} = 2 + \frac{1}{2} = \frac{5}{2},$$
$$a_3 = a_2 + \frac{1}{3!} = \frac{5}{2} + \frac{1}{6} = \frac{16}{6} = \frac{8}{3}, \qquad a_4 = a_3 + \frac{1}{4!} = \frac{8}{3} + \frac{1}{24} = \frac{65}{24},$$
$$a_5 = a_4 + \frac{1}{5!} = \frac{65}{24} + \frac{1}{120} = \frac{326}{120} = \frac{163}{60}.$$