Zadatak 7. Odredi aritmetički niz ako je:

1) $a_1 + a_7 = 42,\ a_{10} - a_3 = 21;$
2) $a_5 + a_{11} = -0.2,\ a_4 + a_{10} = 2.6;$
3) $a_1 + a_5 = 24,\ a_2 \cdot a_3 = 60;$
4) $a_2 + a_3 + a_4 = 3,\ a_1 \cdot a_4 = -20;$
5) $a_2 + a_3 + a_4 + a_5 = 34,\ a_2 \cdot a_3 = 28;$
6) $a_3^2 + a_7^2 = 122,\ a_1 + a_7 = 4.$

Rješenje. **1)**

$$\begin{aligned}
&a_1 + a_7 = 42\\
&\underline{a_{10} - a_3 = 21}\\
&a_1 + a_1 + 6d = 42\\
&\underline{a_1 + 9d - a_1 - 2d = 21}\\
&2a_1 + 6d = 42 \ /\ :2\\
&\underline{7d = 21} \qquad\qquad \Longrightarrow\ d = 3;\\
&a_1 + 3d = 21\\
&a_1 + 9 = 21 \qquad\qquad \Longrightarrow\ a_1 = 12;
\end{aligned}$$

2)

$$\begin{aligned}
&a_5 + a_{11} = -0.2\\
&\underline{a_4 + a_{10} = 2.6}\\
&a_1 + 4d + a_1 + 10d = -\frac{1}{5}\\
&\underline{a_1 + 3d + a_1 + 9d = \frac{13}{5}}\\
&2a_1 + 14d = -\frac{1}{5}\\
&\underline{2a_1 + 12d = \frac{13}{5}}\\
&2d = -\frac{14}{5} \qquad\qquad \Longrightarrow\ d = -\frac{7}{5};\\
&2a_1 + 14\left(-\frac{7}{5}\right) = -\frac{1}{5}\\
&2a_1 = \frac{97}{5} \qquad\qquad \Longrightarrow\ a_1 = \frac{97}{10};
\end{aligned}$$

3)

$$\begin{aligned}
&a_1 + a_5 = 24\\
&\underline{a_2 \cdot a_3 = 60}\\
&a_1 + a_1 + 4d = 24 \ /\ :2\\
&\underline{(a_1 + d)(a_1 + 2d) = 60}\\
&a_1 = 12 - 2d\\
&(12 - 2d + d)(12 - 2d + 2d) = 60
\end{aligned}$$

$$12 - d = 5 \qquad \Longrightarrow \; d = 7;$$
$$a_1 = 12 - 2 \cdot 7 \qquad \Longrightarrow \; a_1 = -2;$$

4)

$$a_2 + a_3 + a_4 = 3$$
$$\underline{a_1 \cdot a_4 = -20}$$
$$a_1 + d + a_1 + 2d + a_1 + 3d = 3 \;/\; :3$$
$$\underline{a_1(a_1 + 3d) = -20}$$
$$a_1 + 2d = 1 \implies a_1 = 1 - 2d$$
$$(1 - 2d)(1 - 2d + 3d) = -20$$
$$(1 - 2d)(1 + d) = -20$$
$$1 - d - 2d^2 = -20$$
$$2d^2 + d - 21 = 0$$
$$2d^2 + 7d - 6d - 21 = 0$$
$$(d - 3)(2d + 7) = 0$$
$$d_1 = 3, \quad d_2 = -\frac{7}{2};$$
$$(a_1)_1 = 1 - 2d_1, \quad (a_1)_1 = -5;$$
$$(a_1)_2 = 1 - 2d_2, \quad (a_1)_2 = 8;$$

5)

$$a_2 + a_3 + a_4 + a_5 = 34$$
$$\underline{a_2 a_3 = 28}$$
$$4a_1 + 10d = 34 \;/\; :2$$
$$\underline{(a_1 + d)(a_1 + 2d) = 28}$$
$$2a_1 = 17 - 5d$$
$$\underline{\left(\frac{17 - 5d}{2} + d\right)\left(\frac{17 - 5d}{2} + 2d\right) = 28}$$
$$\frac{17 - 3d}{2} \cdot \frac{17 - d}{2} = 28$$
$$289 - 68d + 3d^2 - 112 = 0$$
$$3d^2 - 68d + 177 = 0$$
$$d_{1,2} = \frac{68 \pm \sqrt{68^2 - 4 \cdot 3 \cdot 177}}{6} = \frac{68 \pm 50}{6}$$
$$d_1 = \frac{59}{3}, \quad d_2 = 3$$
$$(a_1)_1 = \frac{17 - 5 \cdot \frac{59}{3}}{2} = \frac{17 \cdot 3 - 5 \cdot 59}{6} = \frac{-244}{6} = \frac{-122}{3}$$
$$(a_1)_2 = \frac{17 - 5 \cdot 3}{2} = \frac{17 - 15}{2} = 1$$

6)

$$a_3^2 + a_7^2 = 122$$
$$\underline{a_1 + a_7 = 4}$$
$$(a_1 + 2d)^2 + (a_1 + 6d)^2 = 122$$
$$\underline{2a_1 + 6d = 4 \;/\; :2}$$
$$a_1 = 2 - 3d$$
$$\underline{(2 - 3d + 2d)^2 + (2 - 3d + 6d)^2 = 122}$$
$$(2 - d)^2 + (2 + 3d)^2 - 122 = 0$$
$$4 - 4d + d^2 + 4 + 12d + 9d^2 - 122 = 0$$
$$10d^2 + 8d - 114 = 0 \;/\; :2$$
$$5d^2 + 4d - 57 = 0$$
$$d_{1,2} = \frac{-4 \pm \sqrt{16 + 20 \cdot 57}}{10} = \frac{-4 \pm 34}{10}$$
$$d_1 = 3, \quad d_2 = -\frac{19}{5};$$
$$(a_1)_1 = -7, \quad (a_1)_2 = \frac{67}{5}.$$