Zadatak 4. Ako je s a_n zapisan opći član geometrijskog niza, odredi:

1) a_{10}, ako je $a_3 = 8$, $a_5 = 32$;

2) a_6, ako je $a_2 = \frac{2}{3}$, $a_4 = \frac{8}{27}$;

3) a_9, ako je $a_3 = 4$, $a_5 = 8$;

4) a_4, ako je $a_1 = 1$, $a_7 = \frac{64}{27}$;

5) a_4, ako je $a_3 = \frac{4-2\sqrt{3}}{4}$, $a_5 = \frac{28-16\sqrt{3}}{16}$;

6) a_{11}, ako je $a_3 = \sqrt[6]{2}$, $a_7 = \frac{\sqrt{2}}{2}$.

Rješenje. **1)** $a_3 = 8$, $a_5 = 32 = a_3 \cdot q^2$, $a_{10} = ?$

$$q = \sqrt{\frac{a_5}{a_3}} = \sqrt{\frac{32}{8}} = \sqrt{4} = \pm 2$$

$$a_{10} = a_5 \cdot q^5 = 32 \cdot 2^5 = 2^{10} \quad \text{ili}$$

$$a_{10} = a_5 \cdot q^5 = 32 \cdot (-2)^5 = -2^{10};$$

2) $a_6 = ?$, $a_2 = \frac{2}{3}$, $a_4 = \frac{8}{27}$

$$q = \sqrt{\frac{a_4}{a_2}} = \sqrt{\frac{\frac{8}{27}}{\frac{2}{3}}} = \sqrt{\frac{4}{9}} = \pm\frac{2}{3}$$

$$a_6 = a_4 \cdot q^2 = \frac{8}{27} \cdot \left(\pm\frac{2}{3}\right)^2 = \frac{8}{27} \cdot \frac{4}{9} = \frac{32}{243};$$

3) $a_3 = 4$, $a_5 = 8$, $a_9 = ?$

$$q = \sqrt{\frac{a_5}{a_3}} = \sqrt{\frac{8}{4}} = \pm\sqrt{2}$$

$$a_9 = a_5 \cdot q^4 = 8 \cdot (\pm\sqrt{2})^4 = 8 \cdot 4 = 32;$$

4) $a_1 = 1$, $a_7 = \frac{64}{27}$, $a_4 = ?$

$$a_4 = \sqrt{a_1 \cdot a_7} = \sqrt{1 \cdot \frac{64}{27}} = \pm\frac{8}{3\sqrt{3}} = \pm\frac{8\sqrt{3}}{3};$$

5) $a_3 = \frac{4-2\sqrt{3}}{4}$, $a_5 = \frac{28-16\sqrt{3}}{16}$, $a_4 = ?$

$$a_4 = \sqrt{\frac{4-2\sqrt{3}}{4} \cdot \frac{28-16\sqrt{3}}{16}} = \sqrt{\frac{(\sqrt{3}-1)^2(4-2\sqrt{3})^2}{8^2}}$$

$$= \frac{(\sqrt{3}-1)(4-2\sqrt{3})}{8} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)^2}{8} = \frac{(\sqrt{3}-1)^3}{8} = \left(\frac{\sqrt{3}-1}{2}\right)^3;$$

6) $a_3 = \sqrt[6]{2},\ a_7 = \dfrac{\sqrt{2}}{2},\ a_{11} = ?$

$$a_7 = \sqrt{a_3 \cdot a_{11}} \implies a_{11} = \frac{a_7^2}{a_3} = \frac{\frac{2}{4}}{\sqrt[6]{2}} = \frac{1}{2\sqrt[6]{2}} \cdot \frac{\sqrt[6]{2^5}}{\sqrt[6]{2^5}} = \frac{\sqrt[6]{2^5}}{4};$$

$$a_n = a_1 \cdot q^{n-1},\ a_m = a_1 \cdot q^{m-1}$$

$$a_{\frac{m+n}{2}}^2 = \left(a_1 \cdot q^{\frac{m+n}{2}-1}\right)^2 = a_1^2 q^{m+n-2} = a_1 q^{m-1} a_1 q^{n-1} = a_m a_n$$

$$\implies a_{\frac{m+n}{2}} = \sqrt{a_m a_n}.$$