

Zadatak 5. Dokaži da je niz (a_n) geometrijski:

1) $a_n = -3 \cdot 2^n$;

2) $a_n = \frac{\pi}{3^n}$.

Rješenje.

1) $a_n = -3 \cdot 2^n$ Dokaz provodimo indukcijom po n .

$$a_1 = -3 \cdot 2^1 = -6, a_2 = -3 \cdot 2^2 = -12, q = \frac{a_2}{a_1} = 2.$$

Imamo $a_1 = -3 \cdot 2^1$, $a_2 = -3 \cdot 2^2$. Pretpostavimo da je $a_n = -3 \cdot 2^n$ tada je:

$$a_{n+1} = -3 \cdot 2^{n+1} = -3 \cdot 2^n \cdot 2 = a_n \cdot q.$$

Niz je geometrijski.

2) $a_n = \frac{\pi}{3^n}$;

$$a_1 = \frac{\pi}{3}, a_2 = \frac{\pi}{9}, q = \frac{a_2}{a_1} = \frac{1}{3}.$$

Imamo $a_1 = \frac{\pi}{3} \cdot \left(\frac{1}{2}\right)^0$, $a_2 = \frac{\pi}{3} \cdot \left(\frac{1}{2}\right)^1$. Pretpostavimo da je $a_n = \frac{\pi}{3} \cdot \left(\frac{1}{3}\right)^{n-1}$ tada je:

$$a_{n+1} = \frac{\pi}{3} \cdot \left(\frac{1}{3}\right)^n = \frac{\pi}{3^{n+1}}.$$

Niz je geometrijski.