**Zadatak 41.** Odredi zbroj: $\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\ldots+\left(x^n+\frac{1}{x^n}\right)^2$, $x\neq 0$.

***Rješenje.***

$$\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\ldots+\left(x^n+\frac{1}{x^n}\right)^2=\text{(nakon kvadriranja)}$$

$$=x^2+x^4+\ldots+x^{2n}+\frac{1}{x^2}+\frac{1}{x^4}+\ldots+\frac{1}{x^{2n}}+2n$$

$$=(x^2+x^4+\ldots+x^{2n})+\left(\frac{1}{x^2}+\frac{1}{x^4}+\ldots+\frac{1}{x^{2n}}\right)+2n$$

$$\left(\text{u zagradam su geometrijski nizovi } q_1=x^2,\ (a_1)_1=x^2,\ \ q_2=\frac{1}{x^2},\ (a_1)_2=\frac{1}{x^2}\right)$$

$$=\left(x^2\frac{x^{2n}-1}{x^2-1}\right)+\left(\frac{1}{x^2}\frac{\left(\frac{1}{x^2}\right)^n-1}{\frac{1}{x^2}-1}\right)+2n$$

$$=2n+x^2\frac{x^{2n}-1}{x^2-1}+\frac{1}{x^2}\frac{\frac{1}{x^{2n}}-1}{\frac{1}{x^2}-1}=2n+\frac{x^2(x^{2n}-1)}{x^2-1}+\frac{1}{x^2}\frac{\frac{1-x^{2n}}{x^{2n}}}{\frac{1-x^2}{x^2}}$$

$$=2n+\frac{x^2(x^{2n}-1)}{x^2-1}+\frac{1-x^{2n}}{x^{2n}(1-x^2)}=2n+\frac{x^{2n+2}(x^{2n}-1)+x^{2n}-1}{x^{2n}(x^2-1)}$$

$$=2n+\frac{(x^{2n}-1)(x^{2n+2}+1)}{x^{2n}(x^2-1)}.$$