Rješenja zadataka 2.4

Zadatak 1. Dokaži da je broj 1 limes niza (a_n), $a_n = \dfrac{n+1}{n}$.

Rješenje. Treba pokazati:

$$(\forall \varepsilon > 0)(\exists n_0 \in \mathbf{N})(\forall n \in \mathbf{N})\ n > n_0 \implies \left|\frac{n+1}{n} - 1\right| < \varepsilon$$

$$\left|\frac{n+1}{n} - 1\right| = \left|\frac{n+1-n}{n}\right| = \left|\frac{1}{n}\right| = \frac{1}{n} < \varepsilon \implies n > \frac{1}{\varepsilon}.$$

Dakle,

$$(\forall \varepsilon > 0)(\exists n_0 = \left[\frac{1}{\varepsilon}\right] \in \mathbf{N})(\forall n \in \mathbf{N})\ n > \left[\frac{1}{\varepsilon}\right] \implies \left|\frac{n+1}{n} - 1\right| < \varepsilon.$$