Zadatak 2. Ako je $a_n = \dfrac{1-n}{n+2}$, dokaži da je limes niza (a_n) jednak -1. Za koje je n ispunjeno $|a_n+1| < 0.001$?

Rješenje. $\lim\limits_{n\to\infty} \dfrac{1-n}{n+2} = \lim\limits_{n\to\infty} \dfrac{\frac{1}{n}-1}{1+\frac{2}{n}} = \dfrac{0-1}{1+0} = -1$;

Dokažimo to. Treba pokazati:

$$(\forall\varepsilon>0)(\exists n_0\in\mathbf{N})(\forall n\in\mathbf{N})\ n>n_0 \implies \left|\frac{1-n}{n+2}+1\right| < \varepsilon$$

$$\left|\frac{1-n+n+2}{n+2}\right| = \left|\frac{3}{n+2}\right| = \frac{3}{n+2} < \varepsilon \implies \frac{n+2}{3} > \frac{1}{\varepsilon} \implies n+2 > \frac{3}{\varepsilon} \implies n > \frac{3}{\varepsilon}-2.$$

Dakle,

$$(\forall\varepsilon>0)\left(\exists n_0 = \left[\frac{3}{\varepsilon}-2\right]\in\mathbf{N}\right)(\forall n\in\mathbf{N})\ n > \left[\frac{3}{\varepsilon}-2\right] \implies \left|\frac{1-n}{n+2}+1\right| < \varepsilon.$$

Nađimo sada traženi n za koji vrijedi $|a_n+1| < 0.001$:

$$\begin{aligned}
\left|\frac{1-n}{n+2}+1\right| &< \frac{1}{1000},\\
\left|\frac{1-n+n+2}{n+2}\right| &< \frac{1}{1000},\\
\left|\frac{3}{n+2}\right| &< \frac{1}{1000},\\
\frac{3}{n+2} &< \frac{1}{1000},\\
\frac{n+2}{3} &> 1000,\\
n+2 &> 3000,\\
\implies n &> 2998.
\end{aligned}$$