Zadatak 3. Dokaži da je broj a limes niza (a_n) ako je

1) $a_n=\dfrac{1}{n+2}$, $a=0$; **2)** $a_n=\dfrac{n+2}{n-1}$, $a=1$;

3) $a_n=\dfrac{1}{n^2}$, $a=0$; **4)** $a_n=\dfrac{3-n^2}{n^2+1}$, $a=-1$.

Rješenje. **1)**

$$\left|\frac{1}{n+2}-0\right|=\left|\frac{1}{n+2}\right|=\frac{1}{n+2}<\varepsilon$$

$$\Longrightarrow\ n+2>\frac{1}{\varepsilon}\ \Longrightarrow\ n>\frac{1}{\varepsilon}-2$$

$$\Longrightarrow\ (\forall\varepsilon>0)(\exists n_0=\left[\frac{1}{\varepsilon}-2\right]\in\mathbf{N})(\forall n\in\mathbf{N})\ n>n_0$$

$$\Longrightarrow\ \left|\frac{1}{n+2}-0\right|<\varepsilon;$$

2)

$$\left|\frac{n+2}{n-1}-1\right|=\left|\frac{n+2-n+1}{n-1}\right|=\left|\frac{3}{n-1}\right|=\frac{3}{n-1}<\varepsilon$$

$$\Longrightarrow\ \frac{n-1}{3}>\frac{1}{\varepsilon}\ \Longrightarrow\ n-1>\frac{3}{\varepsilon}\ \Longrightarrow\ n>\frac{3}{\varepsilon}+1$$

$$\Longrightarrow\ (\forall\varepsilon>0)(\exists n_0=\left[\frac{3}{\varepsilon}+1\right]\in\mathbf{N})(\forall n\in\mathbf{N})\ n>n_0$$

$$\Longrightarrow\ \left|\frac{n+2}{n-1}-1\right|<\varepsilon;$$

3)

$$\left|\frac{1}{n^2}-0\right|=\frac{1}{n^2}<\varepsilon\ \Longrightarrow\ n^2>\frac{1}{3}\ \Longrightarrow\ n>\frac{1}{\sqrt{\varepsilon}}$$

$$\Longrightarrow\ (\forall\varepsilon>0)(\exists n_0=\left[\frac{1}{\sqrt{\varepsilon}}\right]\in\mathbf{N})(\forall n\in\mathbf{N})\ n>n_0$$

$$\Longrightarrow\ \left|\frac{1}{n^2}-0\right|<\varepsilon;$$

4)

$$\left|\frac{3-n^2}{n^2+1}+1\right|=\left|\frac{3-n^2+n^2+1}{n^2+1}\right|=\frac{4}{n^2+1}<\varepsilon\ \Longrightarrow\ \frac{n^2+1}{4}>\frac{1}{\varepsilon}$$

$$\Longrightarrow\ n^2+1>\frac{4}{\varepsilon}\ \Longrightarrow\ n^2>\frac{4}{\varepsilon}-1\ \Longrightarrow\ n>\sqrt{\left|\frac{4}{\varepsilon}-1\right|}$$

$$\Longrightarrow\ (\forall\varepsilon>0)(\exists n_0=\left[\sqrt{\left|\frac{4}{\varepsilon}-1\right|}\right]\in\mathbf{N})(\forall n\in\mathbf{N})\ n>n_0$$

$$\Longrightarrow\ \left|\frac{3-n^2}{n^2+1}+1\right|<\varepsilon.$$