

Zadatak 7. Dokaži da su sljedeći nizovi divergentni:

$$1) a_n = (-1)^n \frac{n}{n+1}; \quad 2) a_n = n + (-1)^n;$$

$$3) a_n = (-1)^n n; \quad 4) a_n = \frac{n^2 + 1}{n + 2};$$

$$5) a_n = n + (-1)^n(n - 1).$$

Rješenje.

$$1) a_n = (-1)^n \frac{n}{n+1}; a_1 = -\frac{1}{2}, a_2 = \frac{2}{3}, a_3 = -\frac{3}{4}, a_4 = \frac{4}{5}, a_5 = -\frac{5}{6}, a_6 = \frac{6}{7}, \dots \implies \lim_{k \rightarrow \infty} a_{2k} = 1, \lim_{k \rightarrow \infty} a_{2k-1} = -1 \text{ (dva gomilišta);}$$

$$2) a_n = n + (-1)^n; a_1 = 0, a_2 = 3, a_3 = 2, a_4 = 5, a_5 = 4, a_6 = 7, a_7 = 6, \dots \implies \text{neomeđen niz;}$$

$$3) a_n = (-1)^n n; a_1 = -1, a_2 = 2, a_3 = -3, a_4 = 4, a_5 = -5, a_6 = 6, \dots \implies \text{neomeđen niz;}$$

$$4) a_n = \frac{n^2 + 1}{n + 2}; a_1 = \frac{2}{3}, a_2 = \frac{5}{4}, a_3 = 2, a_4 = \frac{17}{6}, a_5 = \frac{26}{7}, \dots \implies \text{neomeđen niz;}$$

$$5) a_n = n + (-1)^n(n - 1) = \begin{cases} 2n - 1, & n = 2k \\ 1, & n = 2k - 1 \end{cases}; a_1 = 1, a_2 = 3, a_3 = 1, a_4 = 7, a_5 = 1, \dots \implies \text{neomeđen podniz.}$$