Zadatak 9. Izračunaj graničnu vrijednost (limes) sljedećih nizova:

1) $a_n = 1 + \frac{1}{n} + \frac{1}{n^2}$; **2)** $a_n = \frac{1}{n^2} - \frac{2}{n} - 3$;

3) $a_n = \frac{3n^2 - n + 1}{5n^2 + n - 1}$; **4)** $a_n = \frac{n^2 - n + 1}{1 + n - n^2}$;

5) $a_n = \frac{1 - 2n}{3n^2 - n - 2}$; **6)** $a_n = \frac{(n^2 - n + 1)(n^2 - n - 1)}{2n(1 - n^3)}$;

7) $a_n = \frac{1 - 3n}{2 + 3.5n} \cdot \frac{2n^2 - n + 4}{3n - 0.8n^2}$; **8)** $a_n = \frac{n(n+1)(n+2)}{(n+3)(n+4)(n+5)}$;

9) $a_n = \frac{n(n+1)}{(n+2)(n+3)(n+4)}$; **10)** $a_n = \frac{n!}{(n+1)!}$.

Rješenje.

1) $\lim\limits_{n\to\infty}\left(1 + \frac{1}{n} + \frac{1}{n^2}\right) = 1$;

2) $\lim\limits_{n\to\infty}\left(\frac{1}{n^2} - \frac{2}{n} - 3\right) = -3$;

3) $\lim\limits_{n\to\infty}\frac{3n^2 - n + 1}{5n^2 + n - 1} = \lim\limits_{n\to\infty}\frac{3 - \frac{1}{n} + \frac{1}{n^2}}{5 + \frac{1}{n} - \frac{1}{n^2}} = \frac{3}{5}$;

4) $\lim\limits_{n\to\infty}\frac{n^2 - n + 1}{1 + n - n^2} = \lim\limits_{n\to\infty}\frac{1 - \frac{1}{n} + \frac{1}{n^2}}{\frac{1}{n^2} + \frac{1}{n} - 1} = -1$;

5) $\lim\limits_{n\to\infty}\frac{1 - 2n}{3n^2 - n - 2} = \lim\limits_{n\to\infty}\frac{\frac{1}{n^2} - \frac{2}{n}}{3 - \frac{1}{n} - \frac{2}{n^2}} = 0$;

6) $\lim\limits_{n\to\infty}\frac{(n^2 - n + 1)(n^2 - n - 1)}{2n(1 - n^3)} = \lim\limits_{n\to\infty}\frac{(n^2 - n)^2 - 1}{2n - 2n^4} = \lim\limits_{n\to\infty}\frac{n^4 - 2n^3 + n^2 - 1}{2n - 2n^4}$

$= \lim\limits_{n\to\infty}\frac{1 - \frac{2}{n} + \frac{1}{n^2} - \frac{1}{n^4}}{\frac{2}{n^3} - 2} = -\frac{1}{2}$;

7) $\lim\limits_{n\to\infty}\frac{1 - 3n}{2 + 3.5n} \cdot \frac{2n^2 - n + 4}{3n - 0.8n^2} = \lim\limits_{n\to\infty}\frac{\frac{1}{n} - 3}{\frac{2}{n} + \frac{7}{2}} \cdot \frac{2 - \frac{1}{n} + \frac{4}{n^2}}{\frac{3}{n} - \frac{4}{5}} = \frac{-3}{\frac{7}{2}} \cdot \frac{2}{-\frac{4}{5}} =$

$\frac{6}{7} \cdot \frac{10}{4} = \frac{15}{7}$;

8) $\lim\limits_{n\to\infty}\frac{n(n+1)(n+2)}{(n+3)(n+4)(n+5)} = \lim\limits_{n\to\infty}\frac{1\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)}{\left(1 + \frac{3}{n}\right)\left(1 + \frac{4}{n}\right)\left(1 + \frac{5}{n}\right)} = 1$;

9) $\lim\limits_{n\to\infty} \dfrac{n(n+1)}{(n+2)(n+3)(n+4)} = \lim\limits_{n\to\infty} \dfrac{\frac{1}{n}\left(1+\frac{1}{n}\right)}{\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\left(1+\frac{4}{n}\right)} = 0\,;$

10) $\lim\limits_{n\to\infty} \dfrac{n!}{(n+1)!} = \lim\limits_{n\to\infty} \dfrac{1}{n+1} = \lim\limits_{n\to\infty} \dfrac{\frac{1}{n}}{1+\frac{1}{n}} = 0\,.$