

Zadatak 10. Izračunaj limes niza:

$$1) a_n = \frac{2n+3}{3n-1};$$

$$2) a_n = \frac{3n^2+5n+1}{2n^2-n+5};$$

$$3) a_n = \frac{\sqrt{n+1}+n}{\sqrt{n^2+1}+\sqrt{n}};$$

$$4) a_n = \frac{\sqrt{3n^3-2n}+n}{n^2+n};$$

$$5) a_n = \frac{2^n+3^n+0.5^n}{2^n-3^n+0.5^n};$$

$$6) a_n = \frac{2^{n-2}}{2^n-2}.$$

Rješenje.

$$1) \lim_{n \rightarrow \infty} \frac{2n+3}{3n-1} = \lim_{n \rightarrow \infty} \frac{2 + \frac{3}{n}}{3 - \frac{1}{n}} = \frac{2}{3};$$

$$2) \lim_{n \rightarrow \infty} \frac{3n^2+5n+1}{2n^2-n+5} = \lim_{n \rightarrow \infty} \frac{3 + \frac{5}{n} + \frac{1}{n^2}}{2 - \frac{1}{n} + \frac{5}{n^2}} = \frac{3}{2};$$

$$3) \lim_{n \rightarrow \infty} \frac{\sqrt{n+1}+n}{\sqrt{n^2+1}+\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{1}{n} + \frac{1}{n^2}} + 1}{\sqrt{1 + \frac{1}{n^2}} + \sqrt{\frac{1}{n}}} = 1;$$

$$4) \lim_{n \rightarrow \infty} \frac{\sqrt{3n^3-2n}+n}{n^2+n} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{3}{n} - \frac{2}{n^3}} + \frac{1}{n}}{1 + \frac{1}{n}} = 0;$$

$$5) \lim_{n \rightarrow \infty} \frac{2^n+3^n+0.5^n}{2^n-3^n+0.5^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{2}{3}\right)^n + 1 + \left(\frac{1}{6}\right)^n}{\left(\frac{2}{3}\right)^n - 1 + \left(\frac{1}{6}\right)^n} = -1;$$

$$6) \lim_{n \rightarrow \infty} \frac{2^{n-2}}{2^n-2} = \lim_{n \rightarrow \infty} \frac{\frac{1}{4}}{1 - \frac{2}{2^n}} = \frac{1}{4}.$$