

**Zadatak 11.** Odredi limes sljedećih nizova:

$$1) \quad a_n = \frac{\sqrt{n+1}}{\sqrt{n+2}};$$

$$3) \quad a_n = \frac{\sqrt{n^2+1}}{n};$$

$$5) \quad a_n = \sqrt{n+1} - \sqrt{n};$$

$$7) \quad a_n = \frac{\sqrt[3]{n^2+n}}{n+2};$$

$$9) \quad a_n = \frac{\sqrt{n}-2}{n+\sqrt{n}+1};$$

$$2) \quad a_n = \frac{\sqrt{n+1}}{\sqrt{n+1}};$$

$$4) \quad a_n = \frac{\sqrt[3]{2n^3+1}}{\sqrt{2n^2-1}};$$

$$6) \quad a_n = \sqrt{n^2+n-n};$$

$$8) \quad a_n = \sqrt[3]{n} - \sqrt[3]{n+1};$$

$$10) \quad a_n = \sqrt{n^2+5n+1} - \sqrt{n^2-n}.$$

**Rješenje.**

$$1) \quad \lim_{n \rightarrow \infty} \frac{\sqrt{n+1}}{\sqrt{n+2}} = \lim_{n \rightarrow \infty} \sqrt{\frac{n+1}{n+2}} = \lim_{n \rightarrow \infty} \sqrt{\frac{1 + \frac{1}{n}}{1 + \frac{2}{n}}} = 1;$$

$$2) \quad \lim_{n \rightarrow \infty} \frac{\sqrt{n+1}}{\sqrt{n+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{n}}}{1 + \frac{1}{\sqrt{n}}} = 1;$$

$$3) \quad \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+1}}{n} = \lim_{n \rightarrow \infty} \sqrt{\frac{n^2+1}{n^2}} = \lim_{n \rightarrow \infty} \sqrt{1 + \frac{1}{n^2}} = 1;$$

$$4) \quad \lim_{n \rightarrow \infty} \frac{\sqrt[3]{2n^3+1}}{\sqrt{2n^2-1}} = \lim_{n \rightarrow \infty} \sqrt[6]{\frac{(2n^3+1)^2}{(2n^2-1)^3}} = \lim_{n \rightarrow \infty} \sqrt[6]{\frac{4n^6+4n^3+1}{8n^6-12n^4+6n^2-1}} = \\ \lim_{n \rightarrow \infty} \sqrt[6]{\frac{4 + \frac{4}{n^3} + \frac{1}{n^6}}{8 - \frac{12}{n^2} + \frac{6}{n^4} - \frac{1}{n^6}}} = \sqrt[6]{\frac{1}{2}} = \frac{1}{\sqrt[6]{2}};$$

$$5) \quad \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) = \lim_{n \rightarrow \infty} \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = 0;$$

$$6) \quad \lim_{n \rightarrow \infty} (\sqrt{n^2+n} - n) = \lim_{n \rightarrow \infty} \frac{n^2+n-n^2}{\sqrt{n^2+n}-n} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \frac{1}{2};$$

$$7) \quad \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2+n}}{n+2} = \lim_{n \rightarrow \infty} \sqrt[3]{\frac{n^2+n}{n^3+6n^2+12n+8}} = \lim_{n \rightarrow \infty} \sqrt[3]{\frac{\frac{1}{n} + \frac{1}{n^2}}{1 + \frac{6}{n} + \frac{12}{n^2} + \frac{8}{n^3}}} = 0;$$

$$8) \quad \lim_{n \rightarrow \infty} (\sqrt[3]{n} - \sqrt[3]{n+1}) = \lim_{n \rightarrow \infty} \frac{n-n-1}{\sqrt[3]{n^2} + \sqrt[3]{n^2+n} + \sqrt[3]{n^2+2n+1}} = 0;$$

$$9) \lim_{n \rightarrow \infty} \frac{\sqrt{n} - 2}{n + \sqrt{n} + 1} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{1}{n}} - \frac{2}{n}}{1 + \sqrt{\frac{1}{n}} + \frac{1}{n}} = 0;$$

$$10) \lim_{n \rightarrow \infty} \sqrt{n^2 + 5n + 1} - \sqrt{n^2 - n} = \lim_{n \rightarrow \infty} \frac{n^2 + 5n + 1 - n^2 + n}{\sqrt{n^2 + 5n + 1} + \sqrt{n^2 - n}}$$
$$= \lim_{n \rightarrow \infty} \frac{6n + 1}{\sqrt{n^2 + 5n + 1} + \sqrt{n^2 - n}} = \lim_{n \rightarrow \infty} \frac{6 + \frac{1}{n}}{\sqrt{1 + \frac{5}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{1}{n}}} = 3.$$