

Zadatak 12. Odredi limes sljedećih nizova:

$$1) \quad a_n = \frac{\sqrt{n+2}}{\sqrt{n+1} + \sqrt{n+2}};$$

$$2) \quad a_n = \frac{\sqrt{n+1} - n}{\sqrt{n+1} + n};$$

$$3) \quad a_n = \sqrt[3]{n^3 + n^2 + 1} - \sqrt[3]{n^3 - n^2 + 1};$$

$$4) \quad a_n = \sqrt{n + \sqrt{n + \sqrt{n}}} - \sqrt{n};$$

$$5) \quad a_n = \frac{3}{\sqrt{n+3} - \sqrt{n}} - \frac{1}{\sqrt{n+2} - \sqrt{n+1}}.$$

Rješenje.

$$1) \quad \lim_{n \rightarrow \infty} \frac{\sqrt{n+2}}{\sqrt{n+1} + \sqrt{n+2}} = \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{2}{n}}}{\sqrt{1 + \frac{1}{n}} + \sqrt{1 + \frac{2}{n}}} = \frac{1}{2};$$

$$2) \quad \lim_{n \rightarrow \infty} \frac{\sqrt{n+1} - n}{\sqrt{n+1} + n} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{1}{n} + \frac{1}{n^2}} - 1}{\sqrt{\frac{1}{n} + \frac{1}{n^2}} + 1} = -1;$$

$$\begin{aligned} 3) \quad & \lim_{n \rightarrow \infty} (\sqrt[3]{n^3 + n^2 + 1} - \sqrt[3]{n^3 - n^2 + 1}) \\ &= \lim_{n \rightarrow \infty} \frac{n^3 + n^2 + 1 - n^3 + n^2 - 1}{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{(n^3 + n^2 + 1)(n^3 - n^2 + 1)} + \sqrt[3]{(n^3 - n^2 + 1)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{2}{\sqrt[3]{\left(1 + \frac{1}{n} + \frac{1}{n^3}\right)^2} + \sqrt[3]{\left(1 + \frac{1}{n} + \frac{1}{n^3}\right)\left(1 - \frac{1}{n} + \frac{1}{n^2}\right)} + \sqrt[3]{\left(1 - \frac{1}{n} + \frac{1}{n^3}\right)^2}} \\ &= \frac{2}{3}; \end{aligned}$$

$$\begin{aligned} 4) \quad & \lim_{n \rightarrow \infty} \left(\sqrt{n + \sqrt{n + \sqrt{n}}} - \sqrt{n} \right) = \lim_{n \rightarrow \infty} \frac{n + \sqrt{n + \sqrt{n}} + \sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}} + \sqrt{n}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \sqrt{\frac{1}{n}}}}{\sqrt{1 + \sqrt{\frac{1}{n}} + \sqrt{\frac{1}{n^3}}} + \sqrt{1}} = \frac{1}{2}; \end{aligned}$$

$$\begin{aligned} 5) \quad & \lim_{n \rightarrow \infty} \left(\frac{3}{\sqrt{n+3} - \sqrt{n}} - \frac{1}{\sqrt{n+2} - \sqrt{n+1}} \right) = \lim_{n \rightarrow \infty} \left(\frac{3(\sqrt{n+3} + \sqrt{n})}{n+3-n} - \right. \\ & \quad \left. \frac{\sqrt{n+2} + \sqrt{n+1}}{n+2-n-1} \right) \\ &= \lim_{n \rightarrow \infty} (\sqrt{n+3} - \sqrt{n+2} + \sqrt{n} - \sqrt{n+1}) = \lim_{n \rightarrow \infty} \left(\frac{n+3-n-2}{\sqrt{n+3} + \sqrt{n+2}} + \right. \\ & \quad \left. \frac{n-n-1}{\sqrt{n} + \sqrt{n+1}} \right) \end{aligned}$$

$$= \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n+3} + \sqrt{n+2}} - \frac{1}{\sqrt{n} + \sqrt{n+1}} \right) = 0.$$