

**Zadatak 13.** Odredi limes sljedećih nizova:

- 1)  $a_n = \frac{3^n + 2^n}{3^n - 2^n}$ ;
- 2)  $a_n = \frac{3^{n-1}}{3^n - 2}$ ;
- 3)  $a_n = \frac{2^n - 3^n}{4^n - 6^n + 9^n}$ ;
- 4)  $a_n = \frac{2^{1/n} - 1}{2^{1/n} + 1}$ ;
- 5)  $a_n = \sqrt[n]{1 + 2^n}$ ;
- 6)  $a_n = \frac{2^n}{1 + 3^n}$ ;
- 7)  $a_n = \frac{2^n + 3^n}{n^2 + 2 \cdot 3^n}$ ;
- 8)  $a_n = \frac{2 \cdot 3^n + 1}{2 - 5 \cdot 3^n}$ ;
- 9)  $a_n = \left(\frac{1}{2}\right)^n \left(\frac{2}{3}\right)^n \left(\frac{3}{4}\right)^n$ ;
- 10)  $a_n = \frac{3^n + 4^n + 5^n}{2^n + 6^n}$ .

*Rješenje.*

- 1)  $\lim_{n \rightarrow \infty} \frac{3^n + 2^n}{3^n - 2^n} = \lim_{n \rightarrow \infty} \frac{1 + \left(\frac{2}{3}\right)^n}{1 - \left(\frac{2}{3}\right)^n} = 1$ ;
- 2)  $\lim_{n \rightarrow \infty} \frac{3^{n-1}}{3^n - 2} = \lim_{n \rightarrow \infty} \frac{1}{3 - \frac{2}{3^{n-1}}} = \frac{1}{3}$ ;
- 3)  $\lim_{n \rightarrow \infty} \frac{2^n - 3^n}{4^n - 6^n + 9^n} = \lim_{n \rightarrow \infty} \frac{2^n - 3^n}{2^{2n} - 2^n 3^n + 3^{2n}} \cdot \frac{2^n + 3^n}{2^n + 3^n} = \lim_{n \rightarrow \infty} \frac{2^{2n} - 3^{2n}}{2^{3n} + 3^{3n}} =$   
 $\lim_{n \rightarrow \infty} \frac{3^{2n} \left(\left(\frac{2}{3}\right)^{2n} - 1\right)}{3^{3n} \left(\left(\frac{2}{3}\right)^{3n} - 1\right)} = \lim_{n \rightarrow \infty} \frac{\left(\frac{2}{3}\right)^{2n} - 1}{3^n \left(\left(\frac{2}{3}\right)^{3n} - 1\right)} = 0$ ;
- 4)  $\lim_{n \rightarrow \infty} \frac{2^{\frac{1}{n}} - 1}{2^{\frac{1}{n}} + 1} = \frac{1 - 1}{1 + 1} = 0$ ;
- 5)  $\lim_{n \rightarrow \infty} \sqrt[n]{1 + 2^n} = \lim_{n \rightarrow \infty} (1 + 2^n)^{\frac{1}{n}} = \lim_{n \rightarrow \infty} \left[2^n \left(\frac{1}{2^n} + 1\right)\right]^{\frac{1}{n}} = \lim_{n \rightarrow \infty} 2 \cdot \left(1 + \frac{1}{2^n}\right)^{\frac{1}{n}} = 2$ ;
- 6)  $\lim_{n \rightarrow \infty} \frac{2^n}{1 + 3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{2}{3}\right)^n}{\left(\frac{1}{3}\right)^n + 1} = 0$ ;
- 7)  $\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{n^2 + 2 \cdot 3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{2}{3}\right)^n + 1}{\frac{n^2}{3^n} + 2} = \frac{1}{2}$ ;
- 8)  $\lim_{n \rightarrow \infty} \frac{2 \cdot 3^n + 1}{2 - 5 \cdot 3^n} = \lim_{n \rightarrow \infty} \frac{2 + \left(\frac{1}{3}\right)^n}{\frac{2}{3^n} - 5} = -\frac{2}{5}$ ;
- 9)  $\lim_{n \rightarrow \infty} \left(\frac{1}{2}\right)^n \left(\frac{2}{3}\right)^n \left(\frac{3}{4}\right)^n = \lim_{n \rightarrow \infty} \left(\frac{1}{4}\right)^n = 0$ ;

$$10) \lim_{n \rightarrow \infty} \frac{3^n + 4^n + 5^n}{2^n + 6^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{6}\right)^n + \left(\frac{4}{6}\right)^n + \left(\frac{5}{6}\right)^n}{\left(\frac{2}{6}\right)^n + 1} = 0.$$