

Rješenja zadatka 2.6

Zadatak 1. Odredi zbroj članova beskonačnog geometrijskog niza:

1) $2, 1, \frac{1}{2}, \dots$

2) $4, 2\frac{2}{3}, 1\frac{7}{9}, \dots$

3) $1, \frac{1}{1.2}, \frac{1}{1.2^2}, \dots$

4) $\sqrt{2}, \sqrt{\frac{1}{2}}, \frac{1}{4}\sqrt{2}, \dots$

5) $\frac{\sqrt{2}+1}{\sqrt{2}-1}, \frac{1}{2-\sqrt{2}}, \frac{1}{2}, \dots$

6) $\frac{\sqrt{3}+1}{\sqrt{3}-1}, 1, \frac{\sqrt{3}-1}{\sqrt{3}+1}, \dots$

7) $2 + \sqrt{2}, 1 + \sqrt{2}, 1 + \frac{\sqrt{2}}{2}, \dots$

8) $\frac{2+\sqrt{2}}{2-\sqrt{2}}, 1, \frac{2-\sqrt{2}}{2+\sqrt{2}}, \dots$

Rješenje. 1) $2, 1, \frac{1}{2}, \dots, \frac{1}{2^{n-2}}, \dots \implies a_1 = 2, q = \frac{1}{2};$
 $|q| < 1$ pa slijedi

$$S = \frac{a_1}{1-q} = \frac{2}{1-\frac{1}{2}} = 4;$$

2) $4, \frac{8}{3}, \frac{16}{9}, \dots \implies a_1 = 4, q = \frac{\frac{8}{3}}{4} = \frac{2}{3};$
 $|q| < 1$ pa slijedi

$$S = \frac{a_1}{1-q} = \frac{4}{1-\frac{2}{3}} = 12;$$

3) $1, \frac{5}{6}, \left(\frac{5}{6}\right)^2, \left(\frac{5}{6}\right)^3, \dots, \left(\frac{5}{6}\right)^{n-1}, \dots \implies a_1 = 1, q = \frac{5}{6};$
 $|q| < 1$ pa slijedi

$$S = \frac{a_1}{1-q} = \frac{1}{1-\frac{5}{6}} = 6;$$

4) $\sqrt{2}, \sqrt{\frac{1}{2}}, \frac{1}{4}\sqrt{2}, \dots \implies a_1 = \sqrt{2}, q = \frac{\frac{1}{4}\sqrt{2}}{\sqrt{2}} = \frac{1}{2};$
 $|q| < 1$ pa slijedi

$$S = \frac{a_1}{1-q} = \frac{\sqrt{2}}{1-\frac{1}{2}} = 2\sqrt{2};$$

$$5) \frac{\sqrt{2}+1}{\sqrt{2}-1}, \frac{1}{2-\sqrt{2}}, \frac{1}{2}, \dots$$

$$\Rightarrow a_1 = \frac{\sqrt{2}+1}{\sqrt{2}-1}, q = \frac{\frac{1}{2-\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \frac{\sqrt{2}-1}{\sqrt{2}};$$

$|q| < 1$ pa slijedi

$$\begin{aligned} S &= \frac{a_1}{1-q} = \frac{\frac{\sqrt{2}+1}{\sqrt{2}-1}}{1-\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{\sqrt{2}(\sqrt{2}+1)}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} \\ &= \sqrt{2}(2\sqrt{2}+3) = 4+3\sqrt{2}; \end{aligned}$$

$$6) \frac{\sqrt{3}+1}{\sqrt{3}-1}, 1, \frac{\sqrt{3}-1}{\sqrt{3}+1}, \dots$$

$$\Rightarrow a_1 = \frac{\sqrt{3}+1}{\sqrt{3}-1}, q = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3};$$

$|q| < 1$ pa slijedi

$$\begin{aligned} S &= \frac{a_1}{1-q} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}-1}}{1-\frac{\sqrt{3}-1}{\sqrt{3}+1}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}-1}}{\frac{2}{\sqrt{3}+1}} = \frac{4+2\sqrt{3}}{2(\sqrt{3}+1)} \\ &= \frac{2+\sqrt{3}}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{5+3\sqrt{3}}{2}; \end{aligned}$$

$$7) 2+\sqrt{2}, 1+\sqrt{2}, 1+\frac{\sqrt{2}}{2}, \dots$$

$$\Rightarrow a_1 = 2+\sqrt{2}, q = \frac{1+\sqrt{2}}{2+\sqrt{2}} \cdot \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{2+2\sqrt{2}-\sqrt{2}-2}{4-2} = \frac{\sqrt{2}}{2};$$

$|q| < 1$ pa slijedi

$$S = \frac{a_1}{1-q} = \frac{2+\sqrt{2}}{1-\frac{\sqrt{2}}{2}} = \frac{4+2\sqrt{2}}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{12+8\sqrt{2}}{2} = 6+4\sqrt{2};$$

$$8) \frac{2+\sqrt{2}}{2-\sqrt{2}}, 1, \frac{2-\sqrt{2}}{2+\sqrt{2}}, \dots \Rightarrow a_1 = \frac{2+\sqrt{2}}{2-\sqrt{2}}, q = \frac{2-\sqrt{2}}{2+\sqrt{2}};$$

$|q| < 1$ pa slijedi

$$\begin{aligned} S &= \frac{a_1}{1-q} = \frac{\frac{2+\sqrt{2}}{2-\sqrt{2}}}{1-\frac{2-\sqrt{2}}{2+\sqrt{2}}} = \frac{\frac{2+\sqrt{2}}{2-\sqrt{2}}}{\frac{2\sqrt{2}}{2+\sqrt{2}}} = \frac{6+4\sqrt{2}}{4\sqrt{2}-4} = \frac{3+2\sqrt{2}}{2\sqrt{2}-2} \cdot \frac{2\sqrt{2}+2}{2\sqrt{2}+2} \\ &= \frac{14+10\sqrt{2}}{4} = \frac{7+5\sqrt{2}}{2}; \end{aligned}$$