

Zadatak 2. Odredi zbroj geometrijskog reda:

$$1) 1 + \sin \frac{\pi}{6} + \sin^2 \frac{\pi}{6} + \dots$$

$$2) 1 - \cos \frac{\pi}{6} + \cos^2 \frac{\pi}{6} - \dots$$

$$3) 1 - \sin^2 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} - \dots$$

$$4) 1 + \cos^2 \frac{\pi}{12} + \cos^4 \frac{\pi}{12} + \dots$$

Rješenje. 1) $1 + \sin \frac{\pi}{6} + \sin^2 \frac{\pi}{6} + \dots = 1 + \frac{1}{2} + \frac{1}{4} + \dots \implies a_1 = 1, q = \frac{1}{2};$
 $|q| < 1$ pa slijedi

$$S = \frac{a_1}{1 - q} = \frac{1}{1 - \frac{1}{2}} = 2$$

2) $1 - \cos \frac{\pi}{6} + \cos^2 \frac{\pi}{6} - \dots = 1 - \frac{\sqrt{3}}{2} + \frac{3}{4} - \dots \implies a_1 = 1, q = -\frac{\sqrt{3}}{2};$
 $|q| < 1$ pa slijedi

$$S = \frac{a_1}{1 - q} = \frac{1}{1 + \frac{\sqrt{3}}{2}} = \frac{2}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{4 - 2\sqrt{3}}{1} = 4 - 2\sqrt{3};$$

3) $1 - \sin^2 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} - \dots;$

$$\sin^2 \frac{\pi}{8} = \frac{1 - \cos \frac{\pi}{4}}{2} = \frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{2 - \sqrt{2}}{4};$$

$$\implies a_1 = 1, q = \frac{\sqrt{2} - 2}{4};$$

$|q| < 1$ pa slijedi

$$S = \frac{a_1}{1 - q} = \frac{1}{1 - \frac{\sqrt{2} - 2}{4}} = \frac{4}{6 - \sqrt{2}} \cdot \frac{6 + \sqrt{2}}{6 + \sqrt{2}} = \frac{24 + 4\sqrt{2}}{34}$$

$$= \frac{12 + 2\sqrt{2}}{17} = \frac{2}{17}(\sqrt{2} + 6);$$

4) $1 + \cos^2 \frac{\pi}{12} + \cos^4 \frac{\pi}{12} + \dots;$

$$\cos^2 \frac{\pi}{12} = \frac{1 + \cos \frac{\pi}{6}}{2} = \frac{1 + \frac{\sqrt{3}}{2}}{2} = \frac{2 + \sqrt{3}}{4};$$

$$\implies a_1 = 1, q = \frac{2 + \sqrt{3}}{4};$$

$$\sqrt{3} < 2 \implies \frac{2 + \sqrt{3}}{4} < 1, |q| < 1 \text{ pa slijedi}$$

$$S = \frac{a_1}{1 - q} = \frac{1}{1 - \frac{2 + \sqrt{3}}{4}} = \frac{4}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = 8 + 4\sqrt{3} = 4(2 + \sqrt{3}).$$