

Zadatak 4. Zapiši u obliku razlomka:

1) $0.2\dot{7} \cdot 0.1\dot{6}\dot{3}$; 2) $0.3\dot{8} \cdot 0.5\dot{4}\dot{5}$;

3) $\frac{0.2\dot{5}}{0.12\dot{7}}$; 4) $\frac{0.2\dot{2}\dot{7}}{0.\dot{6}\dot{3}}$.

Rješenje. 1) $x = 0.2\dot{7} \cdot 0.1\dot{6}\dot{3}$;

$$\begin{aligned} 0.2\dot{7} &= \frac{2}{10} + \frac{7}{100} + \frac{7}{1000} + \dots = \frac{1}{5} + \frac{7}{100} \left(1 + \frac{1}{10} + \frac{1}{100} + \dots\right) \\ &= \frac{1}{5} + \frac{7}{100} \cdot \frac{1}{1 - \frac{1}{10}} = \frac{1}{5} + \frac{7}{100} \cdot \frac{10}{9} = \frac{1}{5} + \frac{7}{90} = \frac{25}{90} = \frac{5}{18}; \end{aligned}$$

$$\begin{aligned} 0.1\dot{6}\dot{3} &= \frac{1}{10} + \frac{63}{1000} + \frac{63}{100\,000} + \dots = \frac{1}{10} + \frac{63}{1000} \left(1 + \frac{1}{100} + \dots\right) \\ &= \frac{1}{10} + \frac{63}{1000} \cdot \frac{1}{1 - \frac{1}{100}} = \frac{1}{10} + \frac{63}{1000} \cdot \frac{100}{99} = \frac{1}{10} + \frac{63}{990} \\ &= \frac{99 + 63}{990} = \frac{162}{990} = \frac{9}{55}; \end{aligned}$$

$$x = \frac{5}{18} \cdot \frac{9}{55} = \frac{1}{22};$$

2) $0.3\dot{8} \cdot 0.5\dot{4}\dot{5} = x$;

$$\begin{aligned} 0.3\dot{8} &= \frac{3}{10} + \frac{8}{100} + \frac{8}{1000} + \frac{8}{10000} + \dots = \frac{3}{10} + \frac{8}{100} \left(1 + \frac{1}{10} + \frac{1}{100} + \dots\right) \\ &= \frac{3}{10} + \frac{8}{100} \cdot \frac{1}{1 - \frac{1}{10}} = \frac{3}{10} + \frac{8}{100} \cdot \frac{10}{9} = \frac{3}{10} + \frac{8}{90} \\ &= \frac{27 + 8}{90} = \frac{35}{90} = \frac{7}{18} \end{aligned}$$

$$\begin{aligned} 0.5\dot{4}\dot{5} &= \frac{5}{10} + \frac{45}{1000} + \frac{45}{100\,000} + \frac{45}{10\,000\,000} + \dots = \frac{5}{10} + \frac{45}{1000} \left(1 + \frac{1}{100} + \frac{1}{10\,000} + \dots\right) \\ &= \frac{1}{2} + \frac{9}{200} \cdot \frac{1}{1 - \frac{1}{100}} = \frac{1}{2} + \frac{9}{200} \cdot \frac{100}{99} = \frac{1}{2} + \frac{9}{198} = \frac{11 + 1}{22} = \frac{12}{22} = \frac{6}{11} \end{aligned}$$

$$x = \frac{7}{18} \cdot \frac{6}{11} = \frac{7}{33};$$

3) $\frac{0.2\dot{5}}{0.12\dot{7}} = x$;

$$\begin{aligned} 0.2\dot{5} &= \frac{2}{10} + \frac{5}{100} + \frac{5}{1000} + \frac{5}{10000} + \dots = \frac{2}{10} + \frac{5}{100} \left(1 + \frac{1}{10} + \dots\right) \\ &= \frac{1}{5} + \frac{1}{20} \cdot \frac{10}{9} = \frac{1}{5} + \frac{1}{18} = \frac{18 + 5}{90} = \frac{23}{90} \end{aligned}$$

$$0.12\dot{7} = \frac{12}{100} + \frac{7}{1000} + \frac{7}{10000} + \dots = \frac{12}{100} + \frac{7}{1000} \left(1 + \frac{1}{10} + \dots\right)$$

$$= \frac{3}{25} + \frac{7}{1000} \cdot \frac{10}{9} = \frac{3}{25} + \frac{7}{900} = \frac{108+7}{900} = \frac{115}{900} = \frac{23}{180}$$

$$x = \frac{\frac{23}{90}}{\frac{23}{180}} = 2;$$

$$4) \frac{0.2\dot{2}\dot{7}}{0.\dot{6}\dot{3}} = x;$$

$$0.2\dot{2}\dot{7} = \frac{2}{10} + \frac{27}{1000} + \frac{27}{100000} + \dots = \frac{2}{10} + \frac{27}{1000} \left(1 + \frac{1}{100} + \dots\right)$$

$$= \frac{1}{5} + \frac{27}{1000} \cdot \frac{100}{99} = \frac{1}{5} + \frac{3}{110} = \frac{22+3}{110} = \frac{25}{110} = \frac{5}{22}$$

$$0.\dot{6}\dot{3} = \frac{63}{100} + \frac{63}{10000} + \dots = \frac{63}{100} \left(1 + \frac{1}{100} + \dots\right) = \frac{63}{100} \cdot \frac{100}{99} = \frac{7}{11}$$

$$x = \frac{\frac{5}{22}}{\frac{7}{11}} = \frac{5 \cdot 11}{22 \cdot 7} = \frac{5}{14}.$$