

Zadatak 10. Riješi jednađbe:

- 1) $1 + 2x + 4x^2 + \dots + (2x)^n + \dots$
 $= 3.4 - 1.2x, |x| < 0.5;$
- 2) $1 + |x| + |x^2| + |x^3| + \dots = 5, |x| < 1;$
- 3) $\frac{1}{\sqrt{1 + 7x + 49x^2 + \dots}}$
 $+ \frac{1}{\sqrt{1 - 9x + 81x^2 - \dots}} = 2;$
- 4) $2^{x+2} + 2^{x+1} + 2^x + \dots$
 $= 3^{x+3} + 3^{x+2} + 3^{x+1} + \dots$

Rješenje.

- 1) $1 + 2x + 4x^2 + \dots + (2x)^n + \dots = 3.4 - 1.2x,$
 $|x| < 0.5 \implies |q| = 2|x| < 1$ pa imamo:

$$\frac{1}{1 - 2x} = \frac{17}{5} - \frac{6}{5}x$$

$$(1 - 2x)(17 - 6x) = 5$$

$$17 - 34x - 6x + 12x^2 - 5 = 0$$

$$12x^2 - 40x + 12 = 0 \quad / : 4$$

$$3x^2 - 10x + 3 = 0$$

$$(3x - 1)(x - 3) = 0 \implies x = \frac{1}{3};$$

- 2) $1 + |x| + |x^2| + |x^3| + \dots = 5, |x| < 1;$

$$\frac{1}{1 - |x|} = 5 \implies 1 - |x| = \frac{1}{5} \implies |x| = \frac{4}{5} \implies x = \pm \frac{4}{5};$$

- 3)

$$\frac{1}{\sqrt{1 + 7x + 49x^2 + \dots}} + \frac{1}{\sqrt{1 - 9x + 81x^2 - \dots}} = 2$$

$$\frac{1}{\sqrt{\frac{1}{1 - 7x}}} + \frac{1}{\sqrt{\frac{1}{1 + 9x}}} = 2$$

$$\frac{1}{\sqrt{\frac{1}{1 - 7x}}} + \frac{1}{\frac{1}{1 + 9x}} = 2$$

$$\sqrt{1 - 7x} = 2 - \sqrt{1 + 9x}$$

$$1 - 7x = 4 - 4\sqrt{1 + 9x} + 1 + 9x$$

$$-4 - 16x = -4\sqrt{1 + 9x} \quad / : (-4)$$

$$1 + 4x = \sqrt{1 + 9x} \quad / ^2$$

$$16x^2 + 8x + 1 - 9x - 1 = 0$$

$$16x^2 - x = 0$$

$$x(16x - 1) = 0 \implies x_1 = 0, \quad x_2 = \frac{1}{16};$$

$$\begin{aligned} 4) \quad & 2^{x+2} + 2^{x+1} + 2^x + \dots = 3^{x+3} + 3^{x+2} + 3^{x+1} + \dots; \\ & 2^{x+2} \left(1 + \frac{1}{2} + \frac{1}{4} + \dots \right) = 3^{x+3} \left(1 + \frac{1}{3} + \frac{1}{9} + \dots \right) \\ & 2^{x+2} \frac{1}{1 - \frac{1}{2}} = 3^{x+3} \frac{1}{1 - \frac{1}{3}} \\ & 2^{x+2} \cdot 2 = 3^{x+3} \cdot \frac{3}{2} / \cdot 2 \\ & 2^{x+4} = 3^{x+4} \implies x = -4. \end{aligned}$$