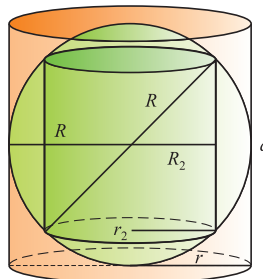


**Zadatak 27.** U jednakostraničan valjak upisana je sfera, sferi je upisan valjak, valjku opet sfera itd. Ako je površina osnog presjeka prvog valjka  $100 \text{ cm}^2$ , koliki je zbroj površina svih sfera?

*Rješenje.*



$r$  – polumjer baze valjka  
 $R$  – polumjer sfere

$$R_1 = r_1, \quad R_2 = r_2, \quad R_3 = r_3, \dots$$

$$P = 100 \implies r_1 = 5, \quad R_1 = 5, \quad \frac{d}{2} = 5, \quad d = 10;$$

Iz slike vidimo:

$$2R_2^2 = R^2 \implies R_2 = \frac{\sqrt{2}}{2}R \implies R_2 = \frac{5\sqrt{2}}{2};$$

Analogno se dobije:

$$\begin{aligned} R_3 &= \frac{\sqrt{2}}{2}R_2 = \left(\frac{\sqrt{2}}{2}\right)^2 R, \\ R_4 &= \frac{\sqrt{2}}{2}R_3 = \left(\frac{\sqrt{2}}{2}\right)^3 R, \\ R_5 &= \frac{\sqrt{2}}{2}R_4 = \left(\frac{\sqrt{2}}{2}\right)^4 R, \dots \end{aligned}$$

pa imamo:

$$\begin{aligned} S &= 4R_1^2\pi + 4R_2^2\pi + 4R_3^2\pi + 4R_4^2\pi + \dots \\ &= 4R^2\pi + 4\left(\frac{\sqrt{2}}{2}\right)^2 R^2\pi + 4\left(\frac{\sqrt{2}}{2}\right)^4 R^2\pi + 4\left(\frac{\sqrt{2}}{2}\right)^6 R^2\pi + \dots \\ &= 4R^2\pi + 4\left(\frac{1}{2}\right)R^2\pi + 4\left(\frac{1}{2}\right)^2 R^2\pi + 4\left(\frac{1}{2}\right)^3 R^2\pi + \dots \\ &= 4R^2\pi \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right) = 4R^2\pi \frac{1}{1 - \frac{1}{2}} = 8R^2\pi = 8 \cdot 25\pi = 200\pi. \end{aligned}$$