



Zadatak 6. Ako je (a_n) aritmetički niz, dokaži da je i niz (b_n) , $b_n = a_{n+1}^2 - a_n^2$ aritmetički.

Rješenje. Tri uzastopna člana novog niza su $a_n^2 - a_{n-1}^2$, $a_{n+1}^2 - a_n^2$, $a_{n+2}^2 - a_{n+1}^2$.
Potrebno je provjeriti relaciju $2(a_{n+1}^2 - a_n^2) = a_n^2 - a_{n-1}^2 + a_{n+2}^2 - a_{n+1}^2$:

$$2(a_{n+1}^2 - a_n^2) = a_n^2 - a_{n-1}^2 + a_{n+2}^2 - a_{n+1}^2$$

$$\iff 3a_{n+1}^2 - 3a_n^2 = a_{n+2}^2 - a_{n-1}^2$$

$$\iff 3(a_{n+1} - a_n)(a_{n+1} + a_n) = (a_{n+2} - a_{n-1})(a_{n+2} + a_{n-1})$$

$$\iff 3d(a_1 + nd + a_1 + (n-1)d) = 3d(a_1 + (n+1)d + a_1 + (n-2)d)$$

$$\iff 2a_1 + (2n-1)d = 2a_1 + (2n-1)d.$$