

Zadatak 9. Dokaži: ako je niz a_1, a_2, \dots, a_n ; $a_i > 0$, aritmetički, onda vrijedi

$$\begin{aligned} \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \\ = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}. \end{aligned}$$

Rješenje.

$$\begin{aligned} S &= \frac{1}{\sqrt{a_2} + \sqrt{a_1}} + \frac{1}{\sqrt{a_3} + \sqrt{a_2}} + \dots + \frac{1}{\sqrt{a_n} + \sqrt{a_{n-1}}} \\ &= \frac{\sqrt{a_2} - \sqrt{a_1}}{a_2 - a_1} + \frac{\sqrt{a_3} - \sqrt{a_2}}{a_3 - a_2} + \dots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}} \\ &= \frac{\sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2} + \dots + \sqrt{a_n} - \sqrt{a_{n-1}}}{d} \\ &= \frac{\sqrt{a_n} - \sqrt{a_1}}{d} = \frac{a_n - a_1}{d(\sqrt{a_n} + \sqrt{a_1})} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}. \end{aligned}$$