Zadatak 10. Ako brojevi $a_1, a_2, \ldots, a_n$, svi različiti od nule, čine aritmetički niz, onda je

$$\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \ldots + \frac{1}{a_{n-1}a_n} = \frac{n-1}{a_1a_n}.$$

Dokaži!

Rješenje. Vrijedi

$$\frac{1}{a_1a_2} = \frac{a_2 - a_1}{a_1a_2} \cdot \frac{1}{d} = \left(\frac{1}{a_1} - \frac{1}{a_2}\right)\frac{1}{d},$$
$$\frac{1}{a_2a_3} = \frac{a_3 - a_2}{a_2a_3} \cdot \frac{1}{d} = \left(\frac{1}{a_2} - \frac{1}{a_3}\right)\frac{1}{d},$$

itd. Zbrojimo taj niz jednakosti i imamo

$$\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \ldots + \frac{1}{a_{n-1}a_n} = \left(\frac{1}{a_1} - \frac{1}{a_n}\right) \cdot \frac{1}{d} = \frac{a_n - a_1}{a_1a_n} \cdot \frac{1}{d} = \frac{n-1}{a_1a_n}.$$