

Zadatak 36.

Izračunaj:

1) $1 + 11 + 111 + \dots + 11 \dots 1$;

2) $2 + 22 + 222 + \dots + 22 \dots 2$;

3) $7 + 77 + 777 + \dots + 77 \dots 7$.

Posljednji broj ima n znamenki.**Rješenje.**

1)

$$\begin{aligned}
 S_n &= \left(1 + \frac{99}{9} + \frac{999}{9} + \dots + \frac{99 \dots 9}{9}\right) \\
 &= \frac{1}{9} (10 - 1 + 10^2 - 1 + 10^3 - 1 + \dots + 10^n - 1) \\
 &= \frac{1}{9} (10 + 10^2 + \dots + 10^n - n) = \frac{1}{9} \left(10 \cdot \frac{10^n - 1}{9} - n\right).
 \end{aligned}$$

2)

$$\begin{aligned}
 S_n &= 2 \cdot \left(1 + 11 + 111 + \dots + 11 \dots 1\right) \\
 &= 2 \cdot \left(1 + \frac{99}{9} + \frac{999}{9} + \dots + \frac{99 \dots 9}{9}\right) \\
 &= \frac{2}{9} (10 - 1 + 10^2 - 1 + 10^3 - 1 + \dots + 10^n - 1) \\
 &= \frac{2}{9} (10 + 10^2 + \dots + 10^n - n) = \frac{2}{9} \left(10 \cdot \frac{10^n - 1}{9} - n\right).
 \end{aligned}$$

3)

$$\begin{aligned}
 S_n &= 7 \cdot \left(1 + 11 + 111 + \dots + 11 \dots 1\right) \\
 &= 7 \cdot \left(1 + \frac{99}{9} + \frac{999}{9} + \dots + \frac{99 \dots 9}{9}\right) \\
 &= \frac{7}{9} (10 - 1 + 10^2 - 1 + 10^3 - 1 + \dots + 10^n - 1) \\
 &= \frac{7}{9} (10 + 10^2 + \dots + 10^n - n) = \frac{7}{9} \left(10 \cdot \frac{10^n - 1}{9} - n\right).
 \end{aligned}$$