

Zadatak 37. Dokaži: $(\underbrace{66\dots6}_n)^2 + \underbrace{88\dots8}_n = 44\dots4$. Pritom imamo n šestica i n osmica te $2n$ četvorki.

Rješenje.

$$\begin{aligned}
 (\underbrace{66\dots6}_n)^2 + \underbrace{88\dots8}_n &= 36 \cdot (\underbrace{11\dots1}_n)^2 + 8 \underbrace{11\dots1}_n \\
 &= 36 \cdot \left(\frac{99\dots9}{9}\right)^2 + 8 \frac{99\dots9}{9} = \frac{36}{81} \cdot (10^n - 1)^2 + \frac{8}{9} (10^n - 1) \\
 &= \frac{4}{9} \cdot (10^n - 1)^2 + \frac{8}{9} (10^n - 1) = \frac{4}{9} \cdot (10^n - 1) (10^n - 1 + 2) \\
 &= \frac{4}{9} \cdot (10^n - 1) (10^n + 1) = \frac{4}{9} \cdot (10^{2n} - 1) \\
 &= \frac{4}{9} \cdot \underbrace{99\dots9}_{2n} = 4 \cdot \underbrace{11\dots1}_{2n} = \underbrace{44\dots4}_{2n}.
 \end{aligned}$$