

Zadatak 38. Koliko je $\sqrt{\underbrace{44\dots 4}_{2n} + \underbrace{11\dots 1}_{n+1} - \underbrace{66\dots 6}_n}$, pri čemu prvi broj ima $2n$ znamenki, drugi $n+1$, a treći n , $n \geq 3$?

Rješenje.

$$\begin{aligned}
 \sqrt{\underbrace{44\dots 4}_{2n} + \underbrace{11\dots 1}_{n+1} - \underbrace{66\dots 6}_n} &= \sqrt{\frac{4}{9}(10^{2n} - 1) + \frac{1}{9}(10^{n+1} - 1) - \frac{6}{9}(10^n - 1)} \\
 &= \sqrt{\frac{4}{9}10^{2n} - \frac{4}{9} + \frac{1}{9}10^{n+1} - \frac{1}{9} - \frac{6}{9}10^n + \frac{6}{9}} \\
 &= \sqrt{\frac{4}{9}10^{2n} + \frac{10}{9}10^n - \frac{6}{9}10^n + \frac{1}{9}} \\
 &= \sqrt{\frac{4}{9}10^{2n} + \frac{4}{9}10^n + \frac{1}{9}} = \sqrt{\left(\frac{2}{3}10^n + \frac{1}{3}\right)^2} \\
 &= \frac{2}{3}10^n + \frac{1}{3} = \frac{6}{9}(10^n - 1) + \frac{6}{9} + \frac{1}{3} \\
 &= \underbrace{66\dots 6}_n + 1 = \underbrace{66\dots 6}_{n-1}7.
 \end{aligned}$$