Zadatak 42. Dokaži da u svakom geometrijskom nizu (a_n) vrijedi:

1) $a_2 + a_4 + a_6 + \ldots + a_{2n} = \dfrac{q}{1+q} S_{2n}$;

2) $\dfrac{1}{a_1} + \dfrac{1}{a_2} + \ldots + \dfrac{1}{a_n} = \dfrac{S_n}{a_1 a_n}$.

Rješenje. **1)** $a_2\,,\ a_4\,,\ a_6\,,\ \ldots,\ a_{2n}$ je geometrijski niz s $\dfrac{2n}{2}$ članova i kvocijentom q^2 te je njegova suma:

$$a_2 + a_4 + a_6 + \ldots + a_{2n} = a_2 \frac{1-(q^2)^n}{1-q^2} = a_1 q \frac{1-q^{2n}}{(1-q)(1+q)}$$

$$= \frac{q}{1+q} a_1 \frac{1-q^{2n}}{1-q} = \frac{q}{1+q} S_{2n};$$

2) Imamo redom:

$$\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n} = \frac{1}{a_1}\left(1 + \frac{1}{q} + \ldots + \frac{1}{q^{n-1}}\right) = \frac{1}{a_1} \cdot \frac{1+q+\ldots+q^{n-1}}{q^{n-1}}$$

$$= \frac{a_1(1+q+\ldots+q^{n-1})}{a_1^2 q^{n-1}} = \frac{S_n}{a_1 a_n}.$$