**Zadatak 47.** Odredi limese nizova s općim članom:

**1)** $a_n = \dfrac{1^2 + 2^2 + \ldots + n^2}{n(n+1)(n+2)}$;

**2)** $a_n = \dfrac{n\sqrt{1+3+5+\ldots+(2n-1)}}{2n^2+n+1}$;

**3)** $a_n = \left(1-\dfrac{1}{2}\right)\left(1-\dfrac{1}{4}\right)\cdots\left(1-\dfrac{1}{2^n}\right)$;

**4)** $a_n = \dfrac{1\cdot 2 + 2\cdot 3 + \ldots + n(n+1)}{1\cdot 3 + 3\cdot 5 + \ldots + (2n-1)(2n+1)}$.

***Rješenje.*** **1)**

$$\lim_{n\to\infty} \frac{1^2 + 2^2 + \ldots + n^2}{n(n+1)(n+2)} = \lim_{n\to\infty} \frac{\frac{1}{6}n(n+1)(2n+1)}{n(n+1)(n+2)}$$
$$= \lim_{n\to\infty} \frac{(2n+4)-3}{6(n+2)} = \lim_{n\to\infty} \left(\frac{1}{3} - \frac{1}{2(n+2)}\right) = \frac{1}{3};$$

**2)**

$$\lim_{n\to\infty} \frac{n\sqrt{1+3+5+\ldots+(2n-1)}}{2n^2+n+1} = \lim_{n\to\infty} \frac{n\sqrt{\frac{n}{2}(1+2n-1)}}{2n^2+n+1}$$
$$= \lim_{n\to\infty} \frac{n^2}{2n^2+n+1} = \lim_{n\to\infty} \frac{n^2+\frac{1}{2}n+\frac{1}{2}-\frac{1}{2}n-\frac{1}{2}}{2n^2+n+1}$$
$$= \frac{1}{2} - \frac{n-1}{4n^2+2n+2} = \frac{1}{2};$$

**3)**

$$\lim_{n\to\infty} \left(1-\frac{1}{2}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{2^n}\right)$$
$$= \lim_{n\to\infty} \left(\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{7}{8}\cdot\ldots\cdot\frac{2^n-1}{2^n}\right) = 0$$

**4)**

$$
\begin{aligned}
&\lim_{n\to\infty}\frac{1\cdot 2+2\cdot 3+\ldots+n(n+1)}{1\cdot 3+3\cdot 5+\ldots+(2n-1)(2n+1)}\\
&\qquad=\lim_{n\to\infty}\frac{(1+1^2)+(2+2^2)+\ldots+(n+n^2)}{(4\cdot 1-1)+(4\cdot 2-1)+\ldots+(4n^2-1)}\\
&\qquad=\lim_{n\to\infty}\frac{(1^2+2^2+3^2+\ldots+n^2)+(1+2+3+\ldots+n)}{4(1+2^2+3^2+\ldots+n^2)-n}\\
&\qquad=\lim_{n\to\infty}\frac{\frac{1}{6}n(n+1)(2n+1)+\dfrac{1}{2}n(n+1)}{4\frac{1}{6}n(n+1)(2n+1)-n}\\
&\qquad=\lim_{n\to\infty}\frac{(n+1)(2n+1)+3(n+1)}{4(n+1)(2n+1)-6}\\
&\qquad=\lim_{n\to\infty}\frac{2n^2+3n+1+3n+3}{8n^2+12n+4-6}\\
&\qquad=\lim_{n\to\infty}\frac{n^2+3n+2}{4n^2+6n-1}=\frac{1}{4}.
\end{aligned}
$$