

Zadatak 50.

Nadi:

$$1) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^{3n+1}; \quad 2) \lim_{n \rightarrow \infty} \left(\frac{3+n}{1+n}\right)^{1-n};$$

$$3) \lim_{n \rightarrow \infty} \left(1 - \frac{1}{5n}\right)^{2n-7}; \quad 4) \lim_{n \rightarrow \infty} \left(\frac{n^2+2}{n^2+1}\right)^{n^2}.$$

Rješenje.

1)

$$\begin{aligned} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^{3n+1} &= \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^{2n} \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^n \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right) \\ &= e \cdot e^{\frac{1}{2}} \cdot 1 = e^{\frac{3}{2}}; \end{aligned}$$

2)

$$\begin{aligned} \lim_{n \rightarrow \infty} \left(\frac{3+n}{1+n}\right)^{1-n} &= \lim_{n \rightarrow \infty} \left(\frac{1+n}{3+n}\right)^{n-1} = \lim_{n \rightarrow \infty} \left(1 - \frac{2}{3+n}\right)^{n-1} \\ &= \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{3+n}\right)^{n+3} \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{3+n}\right)^{-4} \\ &= e^{-2} \cdot \lim_{n \rightarrow \infty} \left(\frac{3+n}{1+n}\right)^4 = e^{-2} \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{2}{1+n}\right)^4 \\ &= e^{-2} \cdot 1 = e^{-2}; \end{aligned}$$

3)

$$\begin{aligned} \lim_{n \rightarrow \infty} \left(1 - \frac{1}{5n}\right)^{2n-7} &= \lim_{n \rightarrow \infty} \left(1 + \frac{-\frac{2}{5}}{2n}\right)^{2n} \cdot \lim_{n \rightarrow \infty} \left(1 - \frac{1}{5n}\right)^{-7} \\ &= e^{-\frac{2}{5}} \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{1}{5n}\right)^7 \\ &= e^{-\frac{2}{5}} \cdot 1 = e^{-\frac{2}{5}}; \end{aligned}$$

4)

$$\begin{aligned} \lim_{n \rightarrow \infty} \left(\frac{n^2+2}{n^2+1}\right)^{n^2} &= \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n^2+1}\right)^{n^2+1} \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n^2+1}\right)^{-1} \\ &= e \cdot \lim_{n \rightarrow \infty} \left(\frac{n^2+1}{n^2+2}\right) = e \cdot \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n^2+2}\right) \\ &= e \cdot 1 = e. \end{aligned}$$