Zadatak 50. Nađi:

1) $\lim\limits_{n\to\infty}\left(1+\frac{1}{2n}\right)^{3n+1}$; **2)** $\lim\limits_{n\to\infty}\left(\frac{3+n}{1+n}\right)^{1-n}$;

3) $\lim\limits_{n\to\infty}\left(1-\frac{1}{5n}\right)^{2n-7}$; **4)** $\lim\limits_{n\to\infty}\left(\frac{n^2+2}{n^2+1}\right)^{n^2}$.

Rješenje. **1)**

$$\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{3n+1}=\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{2n}\cdot\lim_{n\to\infty}\left(1+\frac{\frac{1}{2}}{n}\right)^{n}\cdot\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)$$

$$=e\cdot e^{\frac{1}{2}}\cdot 1=e^{\frac{3}{2}};$$

2)

$$\lim_{n\to\infty}\left(\frac{3+n}{1+n}\right)^{1-n}=\lim_{n\to\infty}\left(\frac{1+n}{3+n}\right)^{n-1}=\lim_{n\to\infty}\left(1-\frac{2}{3+n}\right)^{n-1}$$

$$=\lim_{n\to\infty}\left(1+\frac{-2}{3+n}\right)^{n+3}\cdot\lim_{n\to\infty}\left(1+\frac{-2}{3+n}\right)^{-4}$$

$$=e^{-2}\cdot\lim_{n\to\infty}\left(\frac{3+n}{1+n}\right)^{4}=e^{-2}\cdot\lim_{n\to\infty}\left(1+\frac{2}{1+n}\right)^{4}$$

$$=e^{-2}\cdot 1=e^{-2};$$

3)

$$\lim_{n\to\infty}\left(1-\frac{1}{5n}\right)^{2n-7}=\lim_{n\to\infty}\left(1+\frac{-\frac{2}{5}}{2n}\right)^{2n}\cdot\lim_{n\to\infty}\left(1-\frac{1}{5n}\right)^{-7}$$

$$=e^{-\frac{2}{5}}\cdot\lim_{n\to\infty}\left(1+\frac{1}{5n}\right)^{7}$$

$$=e^{-\frac{2}{5}}\cdot 1=e^{-\frac{2}{5}};$$

4)

$$\lim_{n\to\infty}\left(\frac{n^2+2}{n^2+1}\right)^{n^2}=\lim_{n\to\infty}\left(1+\frac{1}{n^2+1}\right)^{n^2+1}\cdot\lim_{n\to\infty}\left(1+\frac{1}{n^2+1}\right)^{-1}$$

$$=e\cdot\lim_{n\to\infty}\left(\frac{n^2+1}{n^2+2}\right)=e\cdot\lim_{n\to\infty}\left(1-\frac{1}{n^2+2}\right)$$

$$=e\cdot 1=e.$$